

A Comprehensive Look at Financial Volatility Prediction by Economic Variables^{*}

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Abstract

What drives volatility on financial markets? This paper takes a comprehensive look at the predictability of financial market volatility by macroeconomic and financial variables. We go beyond forecasting stock market volatility (by large the focus in previous studies) and additionally investigate the predictability of foreign exchange, bond, and commodity volatility by means of a data-rich modeling methodology which is able to handle a large number of potential predictor variables. We find that volatility in foreign exchange, bond, and commodity markets is predictable by macro and financial predictors both in-sample and out-of-sample. Stock market volatility is less predictable and we only find some in-sample but no out-of-sample evidence of predictability. Interestingly, the best volatility predictors tend to be those that are also useful for predicting returns.

JEL-Classification: G12, G15, G17, C53

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I. Introduction

In this paper we investigate whether financial market volatility is predictable by the information contained in macroeconomic and financial variables. We provide a comprehensive analysis of volatility predictability in foreign exchange, bond, stock, and commodity markets in a data-rich environment that allows for a large number of potential predictors well-known from the return predictability literature (see e.g. [Goyal and Welch, 2003, 2008](#), for a partial overview). Since it is not clear to the economic agent *ex ante* which macro-finance variables are best suited for volatility prediction, we consider an econometric framework which is explicitly suited for dealing with such model uncertainty.¹ We find clear evidence of volatility predictability by macroeconomic and financial variables for foreign exchange, bond, and commodity markets, both from an in-sample and out-of-sample perspective. Results for equities are less pronounced and only indicate some success in in-sample settings but we find no evidence of out-of-sample predictability. Importantly, all our results hold when controlling for the standard autoregressive component of asset return volatility.

Using information in macroeconomic and financial variables to forecast volatility in financial markets is not completely new to the literature but is far from having received the same attention as return predictability (see e.g. [Cochrane and Piazzesi, 2005](#); [Ang and Bekaert, 2007](#); [Ludvigson and Ng, 2009](#); [Lustig, Roussanov, and Verdelhan, 2010b](#), for recent contributions to return predictability).² One early prominent analysis in the literature on the economic determinants of volatility is [Schwert \(1989\)](#) who examines stock market volatility by macro variables and finds little support for volatility predictability. [Paye \(2010\)](#) is a recent study on equity volatility with a similar conclusion. On a more positive note, [Engle, Ghysels, and Sohn \(2008\)](#) analyze the effect of inflation and industrial production growth on stock return volatility, considering each macroeconomic variable separately. They find that

¹See for instance [Avramov \(2002\)](#), [Cremers \(2002\)](#), or [Wright \(2008\)](#) for similar treatments of model uncertainty in financial forecasting setups.

²By contrast, there is a large and growing literature on volatility prediction by pure time-series models (either based upon GARCH models, cf. [Engle \(1982\)](#) and [Bollerslev \(1986\)](#) or based on realized volatility and high-frequency modeling, cf. [Andersen, Bollerslev, Diebold, and Labys \(2003\)](#)) and an enormous literature considering variants thereof. This literature is typically interested in high-frequency movements of volatility, while this paper is mainly interested in low frequency variation.

macro fundamentals do indeed matter for stock return volatility.³

All in all, there is no general conclusion on financial volatility predictability by macroeconomic and financial state variables since most authors employ different sample periods, different forecasting models, different predictors, different evaluation criteria, and almost exclusively focus on stock market volatility. Starting from this observation, we investigate the predictability by macro fundamentals and financial variables of the volatility of four major asset classes (foreign exchange, bonds, stocks, and commodities) in a common econometric framework that allows a multitude of variables to potentially have predictive power. Moreover, we provide a comprehensive analysis by considering a large number of potential macroeconomic and financial explanatory variables instead of just a selected few variables. Most of these explanatory variables are borrowed from the return predictability literature. However, as [Mele \(2007\)](#) argues, countercyclical risk premia in financial markets do not mechanically imply countercyclical return volatility so that it seems worthwhile to investigate sources of volatility predictability separately from return predictability.

Understanding volatility movements is important since it is a consequential input for investment and asset allocation decisions. Moreover, understanding the macroeconomic causes of financial market volatility is interesting in itself since it helps to uncover linkages between price movements in financial markets and underlying risk factors or business cycle state variables.⁴ Furthermore, recent evidence in [Mele \(2008\)](#) and [Fornari and Mele \(2010\)](#) shows that stock market volatility is informative about future business cycle developments so that a better understanding of the driving forces of financial volatility is important for policy makers and monetary authorities.

We provide an additional contribution to the literature on financial volatility, namely that we investigate the properties of out-of-sample predictability by macro-finance variables. While out-of-sample experiments are nowadays fairly standard in the return predictability literature, there seems to be little evidence on this issue for volatility predictability by eco-

³Independent contemporaneous work by [Cakmakli and van Dijk \(2010\)](#) also considers the issue of stock return and volatility predictability based on many (macro-)economic variables. Unlike the Bayesian Model Averaging framework considered here, the authors extract information from macroeconomic series by dynamic factor models.

⁴This is even more important since there is a growing body of evidence that risks associated with volatility are priced in stock, option, bond, and foreign exchange markets (e.g. [Ang, Hodrick, Xing, and Zhang, 2006](#), [Da and Schaumburg, 2009](#), [Menkhoff, Sarno, Schmeling, and Schrimpf, 2010](#), [Christiansen, Rinaldo, and Söderlind, 2010](#) among others). Volatility-based measures have also been shown to predict future stock market returns (see e.g. [Bollerslev, Tauchen, and Zhou, 2009](#)).

nommic variables.⁵ Our paper aims to fill this gap with a comprehensive analysis for four different asset classes.

In our empirical analysis, we investigate a total of 29 potential economic predictors for future volatility movements in the four asset classes mentioned above. We do so by employing a Bayesian model averaging (BMA) approach and compare its forecasting power with an autoregressive benchmark model. For robustness, we also apply a model selection approach which selects the best predictors out of the total pool of variables based on an information criterion (such as the Bayesian information criterion). Although financial volatility is highly autocorrelated in all four asset classes, such that the autoregressive benchmark model already provides a rather good fit, we still find that macroeconomic explanatory variables add significant explanatory power in our forecasting exercises. This is especially true in our out-of-sample tests. Our results are most supportive of volatility predictability in foreign exchange, bond, and commodity markets. For stock market volatility, we also find significant predictive ability in-sample but no economically significant out-of-sample predictability over and above that of the autoregressive component of volatility. In terms of economic effects, it is important to point out that our main forecasting approach shows the strongest predictive ability for predictors that are well-known from the return predictability literature. For example, we find that the term spread matters a lot for bond market volatility and that the default spread and the book-to-market ratio matter most for stock markets. Hence, our results suggest that there are economically meaningful relations between macroeconomic and financial state variables and future volatility of different asset classes. This paper is, to the best of our knowledge, the first to report predictive relations of this kind for a comprehensive set of asset classes. Finally, our results are fairly robust to a number of additional checks and methodological variations.

The remaining part of the paper is structured as follows. Section 2 contains a description of the data. Section 3 describes the econometric framework. Section 4 discusses the empirical results. Section 5 reports the results of several robustness tests. Section 6 concludes.

⁵In independent work, the most recent version of [Paye \(2010\)](#) also contains an out-of-sample analysis of stock market volatility predictability. Relative to [Paye \(2010\)](#), we also consider FX, bond, and commodity market volatility and we explicitly take model uncertainty into account when dealing with a large number of potential predictor variables.

II. Data

We base our analysis on monthly observations of macroeconomic and financial variables and realized volatilities. The sample covers the period from January 1983 to December 2008. Thus, we have 311 monthly time series observations for each variable.

A. Measuring Financial Volatility

The main variables of interest are the financial volatilities which constitute the left hand side variables in our predictive regressions. We use daily returns to calculate monthly realized volatilities. Realized volatility is introduced by [Andersen, Bollerslev, Diebold, and Labys \(2003\)](#) as an accurate proxy for the true, but latent, integrated volatility and is used as observable dependent variable in [Andersen, Bollerslev, Diebold, and Vega \(2003\)](#). The realized variance for asset type i in month t is given as the sum of squared intra-period returns: $\sum_{\tau=1}^{M_t} r_{i;t;\tau}^2$ where $r_{i;t;\tau}$ is the τ th daily continuously compounded return in month t for asset i and M_t denotes the number of trading days during the month t . In our empirical analysis, we define the realized volatility to be the log of the square root of the realized variance since it is better behaved (i.e. closer to normality) than the raw series.

$$RV_{i;t} = \ln \sqrt{\sum_{\tau=1}^{M_t} r_{i;t;\tau}^2}, \quad t = 1, \dots, T. \quad (1)$$

We first outline the construction of our volatility measure for the aggregate FX market. The foreign exchange rates are available from Thomson Financial Datastream. We calculate daily log spot rate changes for the currencies of the following countries all quoted against the USD: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Croatia, Cyprus, Czech Republic, Denmark, Egypt, Euro area, Finland, France, Germany, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Russia, Saudi Arabia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Ukraine, and United Kingdom. Not all currencies are available during the entire sample period, therefore they are included when available. We construct an equally

weighted portfolio consisting of all available currencies at a given point in time, denoted the aggregate FX portfolio. For the aggregate FX portfolio we calculate the time series of the daily spot rate changes and use it to construct realized foreign exchange volatility. We denote the foreign exchange realized volatility at time t by $RV_{FX,t}$ and it can be understood as a measure of global foreign exchange market volatility.⁶

The stock market is represented by the S&P500 futures contract traded on the Chicago Mercantile Exchange (CME). The bond market is represented by the 10-year Treasury note futures contract traded on the Chicago Board of Trade (CBOT).⁷ An advantage of using futures data is that these contracts are highly liquid and therefore transaction costs are of no concern. Moreover, when using futures data we have bond returns straight away without calculating artificial returns from the bond yields. We denote the bond and stock market realized volatilities at time t by $RV_{B,t}$ and $RV_{S,t}$, respectively.

Finally, we employ Standard & Poor’s GSCI commodity index to construct our proxy for commodity market volatility. These data are available from Datastream. In place of the GSCI index, it may have been preferable to use data on the GSCI futures contract as this is actively traded at the CME (see e.g. [Fong and See, 2001](#)). Yet, the GSCI futures only started trading in 1992. Still, the correlation between the realized volatility for the GSCI index and the GSCI futures amounts to 0.97 during the period 1992-2009, so we deem it reasonable to use the GSCI index to obtain a longer time-series. The series of realized volatility at time t in the commodity market is denoted $R_{C,t}$.

[Insert Table 1 about here]

Table 1 shows summary statistics for the four realized volatility series. The average volatilities for commodities and stocks are much larger than the average volatilities for foreign exchange and bonds. The same holds for the standard deviations of the realized volatility

⁶Our approach is similar to [Lustig, Roussanov, and Verdelhan \(2010a\)](#) in that we form an equally weighted portfolio of a large number of currencies against the USD to obtain a “market portfolio” for the global foreign exchange market.

⁷Letting the stock and bond markets be represented by these specific contracts has been done by [Fleming, Kirby, and Ostdiek \(1998\)](#) in a somewhat related setting where they consider volatility linkages between these markets.

series. Normality cannot be rejected except for stock market volatility (based on the Jarque-Bera test). As is well known, realized volatility is highly persistent and we naturally find this behavior for all four asset markets under investigation as indicated by the autocorrelation coefficients.

[Insert Figure 1 about here]

Figure 1 shows the time series of the four volatility series. The time series are highly variable and they do not appear to follow the same time series pattern. This is also reflected in the pair-wise correlation coefficients that are reported in Panel B of Table 1 which are generally not very high in absolute terms. More specifically, bond and stock volatility have a modest correlation of about 37% and all other correlation coefficients are even smaller in absolute magnitude. Thus, we would suspect that the volatility of different asset classes is at least partly driven by different economic variables.

B. Macroeconomic and Financial Predictors

We use a comprehensive set of 29 macroeconomic and financial predictive variables. Our main set of predictors are the explanatory variables used by [Goyal and Welch \(2008\)](#) who employ these variables as predictors of future stock market returns. We then extend the list by further financial and macroeconomic variables which, by economic reasoning, qualify as potential predictors of return volatility for our additional asset classes, namely foreign exchange, bonds, and commodities. The predictive variables are listed below together with their abbreviations in parentheses. An asterisk (*) indicates that the variable is taken from the set of variables studied in [Goyal and Welch \(2008\)](#).

The explanatory variables are: Dividend price ratio* (DP), book to market ratio* (BM), net equity expansion* (NTIS), cross-sectional premium* (CROPR) of [Polk, Thompson, and Vuolteenaho \(2006\)](#), the [Pastor and Stambaugh \(2003\)](#) liquidity factor (LIQ), return on the MSCI World index (MSCI), US market excess return (MKTRF), size factor (SMB), value factor (HML), relative T-bill rate (RTB), relative bond rate (RBR), term spread* (TS), TED spread (TED), long term bond return* (LTR), T-bill rate* (TB), the [Cochrane](#)

and Piazzesi (2005) bond factor (CP), default spread* (DEF), return on the CRB spot index (CRB), return on dollar risk factor (DOL) from Lustig, Roussanov, and Verdelhan (2010a), carry trade factor (CT), FX average bid-ask spread (BAS) as in Menkhoff, Sarno, Schmeling, and Schrimpf (2010), inflation* (INF), industrial production growth (IPGR), orders (ORD), average forward discount (AFD) as in Lustig, Roussanov, and Verdelhan (2010b), M1 growth (M1), investor sentiment (SENT) as in Lemmon and Portniaguina (2006), purchasing manager index (PMI), and housing starts (HS). In sum, we are using nine variables from Goyal and Welch (2008) and 20 additional predictors. Appendix A provides details on the data sources and the construction of the predictive variables.⁸

A few comments on the economic motivation for the different variables are warranted. The list of explanatory variables includes well-known stock market variables such as the dividend price ratio and the book to market ratio which have featured prominently in predictive regressions for stock returns. We also include bond market variables such as the T-bill rate, the term spread and the bond return forecasting factor of Cochrane and Piazzesi (2005). Furthermore, we include potentially relevant predictors from foreign exchange, e.g. the average forward discount which is noteworthy in predictive regressions for future foreign exchange returns as in Lustig, Roussanov, and Verdelhan (2010b) or the TED spread (difference between 3 month LIBOR rate and T-Bill rate) which measures tightening of liquidity in interbank markets as in Brunnermeier, Nagel, and Pedersen (2009). Besides these financial variables some general macroeconomic variables, such as inflation and industrial production growth are included. The latter variable is central in the recent return predictability of excess returns in bonds and foreign exchange (Ludvigson and Ng, 2009; Lustig, Roussanov, and Verdelhan, 2010b).

[Insert Table 2 about here]

Table 2 shows the summary statistics for the set of explanatory variables. Most variables deviate from the normal distribution in terms of skewness and kurtosis and are autocorrelated. Table 2 also shows the correlation between the explanatory variables and each of the four realized volatilities. It is important to notice that it is not the same explanatory

⁸We are most grateful that updated data from Goyal and Welch (2008) are available at Amit Goyal's homepage.

variables that have high contemporaneous correlations (above $|0.40|$, say) with the different realized volatilities. Foreign exchange realized volatility is highly correlated with the dividend yield (DP) and the book to market ratio (BM). Bond volatility is strongly correlated with nominal variables such as the term spread (TS), the Cochrane-Piazzesi bond risk premium factor (CP), and money growth (M1). The stock market realized volatility is strongly correlated with the level of stock market returns (MSCI, MKTRF) and variables indicating market illiquidity (LIQ and TED). The realized volatility of the commodity market, however, is not strongly contemporaneously correlated with any of the explanatory variables.

III. Econometric Framework

We now outline our econometric approach. We employ a framework that enables us to handle a large number of macro-finance variables for forecasting future return volatility.⁹ Note that we use a univariate framework throughout the paper which aims at predicting each of the financial volatilities separately.

We study the predictability of financial volatility using standard predictive regressions for the future realized volatility of asset i

$$RV_{i;t+1} = \alpha + \rho RV_{i;t} + \beta_j' z_{j;t} + u_{t+1}, \quad (2)$$

where β_j denotes the k_j -dimensional vector of regression coefficients on the predictive variables and i indexes foreign exchange (FX), stock (S), bond (B), or commodity (C) return volatility. The subscript j indicates that the composition of the vector of predictive variables $z_{j;t}$ depends on the particular model \mathcal{M}_j . As we have a large number of potentially relevant predictor variables, we investigate $j = 1, \dots, 2^\kappa$ models, where κ denotes the overall number of predictive variables under consideration.

Since volatility is quite persistent, it is necessary to include an AR(1) term $RV_{i,t}$ in the

⁹See e.g. [Avramov \(2002\)](#) and [Ludvigson and Ng \(2009\)](#) for related approaches in the literature on stock return predictability and bond return predictability. [Wright \(2008\)](#) studies the predictability of exchange rates in a data-rich forecasting environment.

predictive regression.¹⁰ Thus, we investigate if there is predictive content of the macroeconomic and financial variables beyond the information that is contained in the time-series history of volatility. We therefore also report results from fitting an AR(1) model for the RV series as the relevant benchmark case. Since the number of potential models is very large (with $29 + 1$ variables we have $2^{30} = 1,073,741,824$ models), it is computationally infeasible to evaluate all possible models analytically.

Given these considerations, we rely on two approaches in this paper. First, we make use of a Bayesian model averaging approach with a stochastic model search algorithm (MC^3). Second, we employ a model selection approach based on different information criteria. We detail these two approaches next.

Bayesian Model Averaging and MC^3 . First, we use a Bayesian model averaging (BMA) approach.¹¹ A particularly attractive feature of the BMA approach is that model uncertainty can be addressed in a coherent way.¹² In our context, model uncertainty refers to the situation that ex ante it is not clear to the economic agent what the important predictive variables are or which combination of variables may be useful for prediction purposes. Unlike the classical approach, Bayesian model averaging does not posit the existence of a true model and is thus particularly suited to deal with a setup where model uncertainty plays a role. Moreover, the BMA approach can be used to obtain optimal weights for forecast combination for the purpose of out-of-sample forecasting. Given the many potential predictor variables, an analytical evaluation of all possible model specifications is not feasible. Hence, we rely on Markov Chain Monte Carlo Model Composition (MC^3) which is a sampling approach drawing from the model space and which is particularly suited for high-dimensional problems such as the one encountered here (cf. [Fernández, Ley, and Steel, 2001](#) or [Koop, 2003](#)).

In a Bayesian framework, one can derive posterior probabilities $p(\mathcal{M}_j|D)$ for each model $j = 1, \dots, 2^\kappa$. These posterior model probabilities, which reflect the usefulness of a particular model after having seen the data D , are used in the BMA framework as weights in a composite

¹⁰[Paye \(2010\)](#) shows that it is necessary to account for persistence for correct inference in predictive regressions for future stock volatility.

¹¹We provide a very brief overview of BMA here. Some further technical details are discussed in Appendix B.

¹²Results in the stock return predictability literature suggest that model uncertainty can be substantial when forecasting returns, see for instance [Avramov \(2002\)](#) or [Schrimpf \(2010\)](#).

model:

$$E[\beta|D] = \sum_{j=1}^{2^\kappa} p(\mathcal{M}_j|D)\beta_j|D, \quad (3)$$

where $\beta_j|D$ denotes the posterior mean of the predictive coefficients in the j th model. Likewise, combined forecasts of BMA can be obtained by weighting the forecasts of the individual models by the posterior model probabilities. Thus, in line with the Bayesian tradition, the data allow us to learn by updating our belief about the quality of a particular model. The posterior model probability is given by

$$p(\mathcal{M}_j|D) = \frac{p(D|M_j)p(\mathcal{M}_j)}{\sum_{i=1}^{2^\kappa} p(D|M_i)p(\mathcal{M}_i)}, \quad (4)$$

where $p(D|M_j)$ is the marginal likelihood and $p(\mathcal{M}_j)$ denotes the prior probability of model j . The expression for the marginal likelihood is obtained as

$$p(D|M_j) = \int p(D|M_j, \beta_j)p(\beta_j|\mathcal{M}_j)d\beta_j, \quad (5)$$

where $p(\beta_j|\mathcal{M}_j)$ is the prior on the parameters of model j and $p(D|M_j, \beta_j)$ is the likelihood of the model.¹³ Because we have a total of 30 potential predictor variables at our disposal (1,073,741,824 models), it is computationally impossible to evaluate all of them analytically. Hence, the results reported below are based on 5,000,000 draws and a burn-in period of 500,000 draws. For further details on the sampling algorithm, we refer to the Appendix.

Model Selection Based on Information Criteria. Second, we use a classical model selection approach based upon information criteria. Unlike the BMA approach, the classical model selection approach neglects that there may be considerable model uncertainty and postulates the existence of a true model. Given the large amount of predictors, we conduct some pretesting before evaluating the models. We reduce the set of potential predictors by only considering variables with a t-statistic greater than two in absolute value in a predictive

¹³We focus on one month forecasting horizons in the following since the Bayesian approach employed here does not allow for longer horizons with overlapping observations in a straightforward way.

regression containing the respective macro variable and the lagged dependent variable. This way, we end up with a set of predictor variables that is not greater than approx. 12 such that an analytical evaluation of all models is computationally feasible.¹⁴ All models are analyzed analytically and then sorted by the Schwarz information criterion (BIC). As is well-known, the BIC favors models that provide a good fit while at the same time penalizing highly parameterized models. Our reported tables include the best five model specifications according to the BIC and we report coefficients, robust standard errors, bootstrap p-values, and the adjusted R^2 .¹⁵

Out-of-Sample Evaluation. We also evaluate our models in an out-of-sample context. As a general rule, we always evaluate the out-of-sample performance of the forecast against the benchmark forecast which is obtained from an AR(1) model. We basically employ the same procedure as in our in-sample tests but we now estimate our models recursively using an expanding window and evaluate the resulting out-of-sample forecasts. More specifically, we start with a 10 year initialization period, estimate predictive regressions in the same way as above to produce the first out-of-sample forecast. We then expand the estimation window and repeat the above steps to obtain out-of-sample forecasts for the next period, and continue this way. In the following, we denote the forecast of our macro-finance augmented model by $f_{i,t+1}^M$ and the forecast of our benchmark model by $f_{i,t+1}^B$.

More specifically, we report Theil's U (TU) which is the root mean square error (RMSE) of our macro-augmented model relative to the RMSE of the benchmark model such that a value smaller than one indicates that the model beats the benchmark in terms of forecast accuracy. In addition, we report out-of-sample R^2 s as in [Campbell and Thompson \(2008\)](#). The out-of-sample R^2 is computed as

$$R_{OOS}^2 = 1 - \frac{\sum_{t=R}^{T-1} (RV_{i,t+1} - f_{i,t+1}^M)^2}{\sum_{t=R}^{T-1} (RV_{i,t+1} - f_{i,t+1}^B)^2} \quad (6)$$

where T denotes the overall sample size, and R is the initialization period which is set to 10 years in our case.

¹⁴This is a common approach and in the spirit of *Occam's razor*, which is also used by e.g. [Ludvigson and Ng \(2009\)](#) in the context of bond return predictability.

¹⁵The bootstrap p-values are computed using a parametric bootstrap along the lines of [Kilian \(1999\)](#) assuming an autoregressive structure for the predictive variables (cf. [Rapach and Wohar, 2006](#)).

Besides these purely descriptive forecast evaluation criteria, we provide bootstrap based statistical inference in order to assess if models augmented by macro-finance predictors are able to significantly outperform the benchmark forecast. An alternative to bootstrap inference could be to rely on asymptotically valid tests in the spirit of the seminal tests by [Diebold and Mariano \(1995\)](#). Since the benchmark model, i.e. the AR(1) is nested by the model of interest, the asymptotic test put forth by [Clark and West \(2007\)](#) may be used. However, the theoretical setup considered in [Clark and West \(2007\)](#) does not cover our case where the forecasts are generated by forecast combination and where a model search over a large amount of models is conducted. Hence, we prefer to rely on a bootstrap approach instead of asymptotic tests.¹⁶ We provide a brief description of our bootstrap procedure in Appendix B.3.

Finally, we report Mincer-Zarnowitz (MZ) tests of unbiased forecasts (by regressing actual realized volatility on a constant and the f^M volatility forecast) and report results for a Wald test of a zero intercept and unit slope coefficient. Our results are based on the GLS version of the Mincer-Zarnowitz test suggested by [Patton and Sheppard \(2009\)](#).

IV. Empirical Results

We now report empirical results on volatility predictability by macroeconomic and financial state variables. We first document in-sample results before moving on to the out-of-sample setting.

A. In-Sample Analysis

Bayesian Model Averaging. We first rely on the BMA approach to examine in-sample volatility predictability and report results for realized volatility in foreign exchange, bond, stock, and commodity markets in Table 3 Panel A-D, respectively. The table presents the ten best predictor variables in terms of posterior probability of inclusion ($\pi|D$). The posterior probability of inclusion reflects the belief on how likely a variable is included in the model after having seen the data. We start from a prior probability of inclusion π of 0.5, which

¹⁶We are grateful to Todd E. Clark for this suggestion. In a similar vein, [Wright \(2008\)](#) relies on a bootstrap to evaluate the out-of-sample superiority of BMA generated forecasts.

implies that every model is deemed equally likely a priori. Hence, if $\pi|D$ exceeds 0.5, our belief of the usefulness of a particular economic variable as a predictor of volatility has been revised upwards in the light of the data evidence. In addition, we report posterior means and standard deviations and Bayesian t-ratios. The latter t-ratios incorporate adjustments for model uncertainty and are thus not comparable to classical t-statistics. We also indicate by 1, if a specific predictor variable appears in the top 5 models according to the posterior model probability $p(\mathcal{M}_j|D)$.

[Insert Table 3 about here]

As a first observation, we find that many macro-finance variables are included in the top five models and/or have a posterior probability exceeding 0.5 so that it is useful to add this information when forecasting volatility. However, there are clearly only few economic variables that can be considered as truly robust predictors after accounting for model uncertainty which renders many economic and financial predictors insignificant (when judged according to the Bayesian t-ratios). As expected, the autoregressive component is important for all four volatilities and is always included in the list of best predictors.

It is striking that the top predictor variables, i.e. the variables with the highest posterior probabilities of inclusion (above 0.50) and generally the highest Bayesian t-ratios, are usually those that have a clear and economic link to the market under study. Thus, the important explanatory variables for the volatility differ across asset class.

We find that the average forward discount (AFD) (and – to a lesser extent – the TED spread) is quite important in the FX market which squares well with findings for FX return predictability in [Lustig, Roussanov, and Verdelhan \(2010b\)](#). The most important bond market volatility predictors is the term spread (TS) which is a well-known bond return predictor (e.g. [Campbell and Shiller, 1991](#)). We find it interesting that the level of the term spread also forecasts bond market volatility and that it does so significantly with a Bayesian t-ratio of more than two which takes model uncertainty into account. Another bond related variable, namely the default spread (DEF) also helps in predicting bond volatility. Next, we find that the default spread (DEF) and book-to-market ratio (BM) are most important for stock market volatility and also significant in terms of their Bayesian t-ratios. These two variables are also known to be related to future stock returns so that it seems interesting

that they also matter most for stock return volatility. In addition, the T-bill rate (TB) and the US market excess return (MKTRF) are important variable for predicting stock market volatility. The most influential variables for future commodity volatility are the T-bill rate (TB), the term spread (TS), and the default spread (DEF). Thus, the commodity volatility is mainly influenced by bond market related variables.

In sum, the results suggest a close economic link between return and volatility predictability in these asset classes.

Information Criteria. Table 4 shows the in-sample results based on information criteria. The five top-performing models (i.e. the models with the lowest BIC among the investigated set of models) as well as the AR(1) benchmark are tabulated.

[Insert Table 4 about here]

Overall, the macroeconomic variables add significant in-sample predictive ability for all four asset classes. As shown by the table, the macroeconomic variables provide additional explanatory power beyond the AR(1) benchmark model for the foreign exchange volatility. The adjusted R^2 of the AR(1) model is 0.356 whereas it is 0.412 for the top-performing model based on macro-finance predictors. Likewise, in the case of bond market volatility the AR(1) is also outperformed by the model based on macro-finance variables. Note that the improvement from the macro-finance variables is not quite as strong when it comes to forecasting commodity market volatility. Furthermore, the improvements offered by including the economic variables is even smaller when considering stock market volatility which is in line with previous results by [Paye \(2010\)](#).

As expected, it is again clearly visible that the autoregressive component is important for all four volatilities. The lagged realized volatility is included in all the top-performing models. There is not much difference between the predictive power of the five top-performing models when measured by their adjusted R^2 s, however. Thus, it appears that it is not overly important which of the top models is actually applied for forecasting purposes. This also means, that from a pure forecasting perspective, it may not so much be a question of which macroeconomic variables are applied but whether macroeconomic variables are considered

at all.¹⁷

Interestingly, it is not the same macroeconomic variables that have predictive content for all four asset classes. Rather, there are notable differences between the most important predictors of FX, bond, stock, and commodity volatility, respectively. We often find classic predictors of returns of one particular asset class to matter for volatility of another asset class: For instance, the average forward discount (AFD) and the carry trade return (CT) show up as prominent predictors for stock market volatility. A stock return predictor, net equity expansion (NTIS) matters for future bond volatility. The dividend price ratio (DP), and the size factor (SMB) are important for commodity volatility. The [Cochrane and Piazzesi \(2005\)](#) bond factor (CP) matters for foreign exchange return volatility. While these relations seem hard to interpret directly in economic terms, they seem to corroborate our claim above that it is not so much a question of which macro-finance variables to include for in-sample forecasting with a classical approach but rather the question whether these variables are included at all or not since the model selection algorithm just extracts the necessary forecasting information from the set of correlated predictor variables.

By contrast, the BMA approach provides much more economically intuitive results. On these grounds, we favor the BMA approach to relying on information criteria for in-sample forecasting. Still, the results from the two approaches are not too different when it comes to their raw forecasting performance: Both approaches have strong autoregressive components and both approaches show that macroeconomic and financial variables improve the predictive ability when forecasting volatility in financial markets when measured by the predictive R^2 .

B. Out-of-Sample Analysis

We proceed by investigating the out-of-sample predictive power of our macro-finance variables for future financial volatility. This exercise is especially important since it is interesting to know whether market participants could usefully employ macro-finance information to improve their volatility forecasts in a real-time setting. Furthermore, there is little earlier evidence in the literature on this topic as discussed above.

¹⁷There is a large literature on the optimal combination of forecasts and how to conduct forecasting with many predictors, cf. [Stock and Watson \(2004\)](#) and [Timmermann \(2006\)](#) for surveys. In our out-of-sample forecast analysis we investigate different types of combination approaches.

Summary statistics for the evaluating out-of-sample forecasts are reported in Table 5. Table 5 shows the out-of-sample results based upon three variants of forecast combination obtained from the MC^3 sample algorithm: (i) forecasts based on the model which takes the highest posterior model probability when the forecast is made, (ii) combining forecasts according to Bayesian model averaging, and (iii) the equally weighted forecast of the 10 best models (in terms of posterior model probability) at the time of the forecast.¹⁸

First, in terms of methodology, we find that the out-of-sample results are fairly similar across combination methods and are therefore robust to the exact way they are generated. Next, in terms of forecasting performance, we find clear evidence of out-of-sample predictability for foreign exchange, bond, and commodity volatility. For example, Theil’s U is uniformly below one for all model selection and weighting schemes for all three asset classes and the monthly out-of-sample R^2 s (see Campbell and Thompson, 2008) are always positive and reach values of 8.3% for foreign exchange and 10.5% for commodity volatility.

[Insert Table 5 about here]

Table 5 also includes bootstrap p-values to test if the macro-finance augmented out-of-sample forecasts outperform the benchmark in terms of mean square forecast errors. We rely on a model-based wild bootstrap imposing the null of no predictability by macro-finance variables as described in section 3 and Appendix B.3. These bootstrap p-values ($\#TU^{bs} < TU$) are computed as the proportion of Theil’s U statistics in the artificial bootstrap samples that are smaller than the sample Theil’s U. Thus, these p-values are one-sided and test the null of equal predictive performance against the alternative of superior performance of the model including macro-finance predictors against the benchmark. Our results show substantial improvements by macro-finance augmented models relative to the benchmark for volatility prediction in FX, bond and commodity markets in that root mean square prediction errors are significantly smaller than those of the benchmark model. By contrast, the results for stock market volatility indicate poor out-of-sample predictability when judged by Theil’s U or R^2_{OOS} , which is also corroborated by the bootstrap results. Thus, we find that stock market volatility is not well predictable by economic variables in an out-of-sample setting in

¹⁸The out-of-sample forecasts are based on a ten year initialization period and an expanding window. For computational reasons, the number of Monte-Carlo draws for the MC^3 algorithm in the out-of-sample exercise is set to 1,000 as opposed to the in-sample results which are obtained with 5 million draws.

line with recent findings in [Paye \(2010\)](#). Finally, the Mincer-Zarnowitz restrictions indicate biased forecasts for foreign exchange and commodity volatility forecasts but not for bond volatility forecasts.

Overall, we find the strongest evidence for out-of-sample volatility predictability for commodity and foreign exchange markets. Bond market volatility still appears predictable out-of-sample – but to a lesser degree – whereas stock market volatility appears to be unpredictable by macro-finance predictors in this setting.

Dynamic Out-of-Sample Performance. We investigate the dynamics of out-of-sample predictability by means of Net-SSE plots for the BMA approach and plot results for all four asset classes in Figure 2. The figure is based on Net-SSE plots as in [Goyal and Welch \(2003, 2008\)](#) and shows the cumulative sum of squared forecast errors of the benchmark model (the AR(1)) minus the squared errors of a forecast model based on economic variables. Hence, a positive slope indicates a superior performance of the macro-finance augmented model relative to the benchmark at a particular point in time.

[Insert Figure 2 about here]

In line with our out-of-sample results, we find strong out-of-sample predictability for FX and commodity markets, less predictability in an out-of-sample setting for the bond market (although we still find a positive Net-SSE), and no out-of-sample predictability for stock market volatility. More important, however, is the finding that there does not seem to be a common trend in all four asset classes such that it may have become increasingly difficult or less difficult to forecast volatility which would indicate a major shift in these four markets in general. Thus, we document that the out-of-sample predictability of financial volatility is time-varying which is in line with the results for return predictability in [Goyal and Welch \(2008\)](#) and [Timmermann \(2008\)](#).

Summing up so far, we find that financial volatility in foreign exchange, bond, and commodity markets is clearly predictable and that augmenting the classic AR(1) benchmark for volatility prediction by macro-finance variables well known from the earlier literature adds incremental value. Importantly, these results hold when considering in-sample forecasts and

they continue to hold in a realistic out-of-sample forecasting setting for all markets except stock volatility. Moreover, the results are fairly stable across estimation methods.

V. Robustness Analysis

A. Nonlinearities

As an additional analysis, we report results for an extended specification where we allow for predictors in levels as well as in squared terms (we take squares of de-meanned time-series to capture large deviations from their averages). We include these squared terms to account for potential non-linearities in the predictive ability of macro variables, since what may matter for future volatility could also be unusually large movements of our predictor variables and not only the level of the predictor per se.¹⁹

[Insert Table 6 about here]

[Insert Table 7 about here]

Table 6 shows the in-sample results. Inspection of the table reveals that allowing for squared terms in addition to predictors in levels hardly affects our conclusions of which variables are important predictors for the four types of asset volatility. Table 7 (Panel A) report the corresponding out-of-sample results based upon the MC^3 algorithm. As shown by the tables, out-of-sample predictability is actually worsened in several instances. So, there is no evidence that including nonlinearities improves the out-of-sample predictability of the macro-finance augmented model.

Overall, we find no evidence that (simple) nonlinearities in our macro-finance variables are of any importance for predicting financial volatility, neither in-sample nor out-of-sample.

¹⁹Also see Ludvigson and Ng (2009) who account for non-linearities by considering squared (and cubic) terms.

B. Absolute Values

As a further robustness check, we employ absolute values of our predictors since it may well be that it is not the level of a predictor variable that forecasts volatility but rather the fact whether a predictor variable takes on an “extreme” value that matters for volatility. For example, extremely low (high) values of the book to market ratio (BM) today, i.e. a high market valuation relative to fundamentals, should signal relatively low (high) stock returns in the future so that the effects on volatility may be ambiguous. To this end, we use absolute values for de-meaned variables of our predictors to predict volatility in this robustness test.

[Insert Table 8 about here]

The in-sample results are shown in Table 8. As shown by the table, the empirical results based on absolute values of the predictors are different from the results based upon the level of the explanatory variables in that different explanatory variables have high posterior probabilities. The top predictor variables change when using absolute values instead of the variable itself. The return on dollar risk factor (DOL) now shows up as the only important explanatory variable in the top FX models. The important macroeconomic variables also change for the bond volatility, now it is relative bond rate (RBR), T-bill rate (TB), and carry trade factor (CT). In the case of stock market volatility none of the macroeconomic variables appears important as there are none with posterior probability of inclusion greater than 0.50. For predicting commodity volatility only inflation (INF) in absolute terms is important.

Table 7 (Panel B) shows the out-of-sample results from using absolute values. Absolute values of the predictor variables hardly have any out-of-sample predictive power. Thus, the conclusion from this out-of-sample exercise is that neglecting the information in the sign of the level of the explanatory variables and using only their absolute values clearly worsens the out-of-sample predictability of financial volatility. Most Theil’s U measures and out-of-sample R^2 s are negative and there is no evidence for volatility predictability by absolute values based on our bootstrap p-values.

Overall, the results from these robustness analyses suggest that the level and the sign of the macro-finance variables provides important information for future volatility. Ignoring this information does not help from a forecasting perspective.

VI. Conclusion

In this paper we provide a comprehensive analysis of volatility predictability in financial markets by macroeconomic and financial predictors. Compared to the previous literature, we extend the analysis in three directions. First, not only do we consider predicting stock market volatility but also the volatility of four main asset classes, namely foreign exchange, bonds, stocks, and commodities. Second, we allow for a comprehensive set of predictive variables which goes far beyond existing studies in the literature on volatility predictability and use model selection and forecast combination procedures to assess whether volatility in financial markets can be predicted by economic variables. Third, we investigate both in-sample and out-of-sample predictability, the latter being new to this line of literature. We use various empirical estimation strategies drawing on a methodology designed for data-rich forecasting environments and show that the results are fairly consistent across these.

We find, not surprisingly, that financial volatility is highly autocorrelated, and therefore lagged volatility is an important predictor variable. Nevertheless, we show in this paper that economic variables provide additional information about future volatility for all four asset classes in an in-sample setting. In line with the earlier literature (e.g. [Schwert, 1989](#); [Paye, 2010](#)) we find little evidence of economically meaningful out-of-sample predictability of stock market return volatility. By contrast, our results are supportive of volatility predictability by macroeconomic and financial variables in foreign exchange, bond, and commodity markets in a realistic out-of-sample setting.

Finally, our paper underscores that results for stock market results are quite different from results for FX, bond, and commodity markets. Thus, empirical stylized facts for stock market volatility predictability cannot easily be transferred to other financial instruments. It would be interesting to learn more about why the results are so different across asset classes in future research.

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Appendix

A. Data Sources and Description

[Insert Table [A.1](#) about here]

B. Methodological Details

In this appendix, we provide some additional details on the Bayesian methods that are underlying the results discussed in the main text. We first describe the elicitation of prior distributions in the Bayesian Model Averaging (BMA) setup. We then provide some details on the Markov Chain Monte Carlo Model Composition algorithm (MC^3) which is used for sampling from the set of models $\mathcal{M}_1, \dots, \mathcal{M}_{2^\kappa}$.

B.1 Prior Elicitation

For ease of exposition, we denote the dependent variable as Y , which is a $T \times 1$ vector of realized volatility as in Eq. (2). The predictive variables are collected in a matrix Z_j which has dimension $T \times k_j$ depending on the particular model \mathcal{M}_j . We are considering a linear regression model with i.i.d. errors which are assumed to be normal with mean zero and variance σ^2 . It is common in the BMA setup to work with the strict exogeneity assumption of the regressors such that a closed form expression for the likelihood can be derived (cf. [Wright, 2009](#)).²⁰

We follow most of the extant BMA literature and choose to work with a natural conjugate prior distribution for the model parameters $p(\beta_j|\mathcal{M}_j)$ and $p(\sigma^2)$. Thus, our prior on the predictive coefficients β_j conditional on σ^2 is taken to be a normal distribution

$$\beta_j|\sigma^2 \sim \mathcal{N}(0, \sigma^2 \phi(Z_j' Z_j)^{-1}), \quad (\text{A.1})$$

²⁰Of course, in a time-series setup as the one considered here, strict exogeneity is typically violated. Nevertheless, given that this violation is generally considered to be of minor relevance for the forecasting problem, the literature [Stock and Watson \(2004\)](#) and [Wright \(2009\)](#) generally assumes strict exogeneity, which provides an elegant theoretical framework for model averaging.

which is centered around zero, i.e. it is expected a-priori that there is no predictive power by the economic variables.²¹ The prior on the predictive coefficients is proper – an important feature to obtain meaningful Bayes factors for model comparison – but it is relatively uninformative, where the amount of informativeness is controlled by the ϕ hyperparameter. The prior on σ^2 is a standard improper prior, proportional to $1/\sigma^2$.

Given these assumptions, the expression for the marginal likelihood takes the following form

$$p(D|\mathcal{M}_j) \propto (1 + \phi)^{-k_j/2} S_j^{-T}, \quad (\text{A.2})$$

where $S_j^2 = Y'Y - Y'Z_j(Z_j'Z_j)^{-1}Z_j'\frac{\phi}{1+\phi}$. The expression in (A.2) is important since it enters Eq. (4) and thus plays an essential role for the computation of posterior model probabilities $p(\mathcal{M}_j|D)$. Given the likelihood and the prior, the posterior mean of the predictive coefficients takes the form

$$\beta_j|D = \frac{\phi}{1 + \phi} (Z_j'Z_j)^{-1} Z_j'Y. \quad (\text{A.3})$$

In this BMA setup there are two modeling choices which require input by the researcher. First, the hyperparameter ϕ must be selected, which controls the degree of informativeness of the prior on the predictive coefficients. A higher ϕ means a less informative prior (i.e. a higher prior variance), whereas a lower ϕ (approaching zero) induces more shrinkage towards the no-forecastability case. We select the ϕ hyperparameter according to the simulation-based recommendations in [Fernández, Ley, and Steel \(2001\)](#). The second choice is that we assign equal prior probability on the models, i.e. we take $1/2^\kappa$ as the prior model probability $p(M_j)$. This implies a prior probability of inclusion for each predictive variable of $\pi = 1/2$.

B.2 MC³ Algorithm

The MC³ algorithm is a Markov Chain Monte Carlo method of sampling from the distribution of models and has similarities with a Metropolis-Hastings algorithm. For each run r of the algorithm, a candidate model M^* is drawn from the model space $\mathcal{M}_1, \dots, \mathcal{M}_{2^\kappa}$ which can either be accepted – if it improves on the model drawn in the previous draw $M^{(r-1)}$ – otherwise it is rejected. If the drawn model is rejected then the chain remains at the previous model $M^{(r-1)}$. The acceptance

²¹This prior specification is also known as a so-called g-prior framework and is originally due to [Zellner \(1986\)](#).

probability $\Xi(M^{(r-1)}, M^*)$ is expressed as

$$\Xi(M^{(r-1)}, M^*) = \min \left\{ \frac{p(D|M^*)p(M^*)}{p(D|M^{(r-1)})p(M^{(r-1)})}; 1 \right\}, \quad (\text{A.4})$$

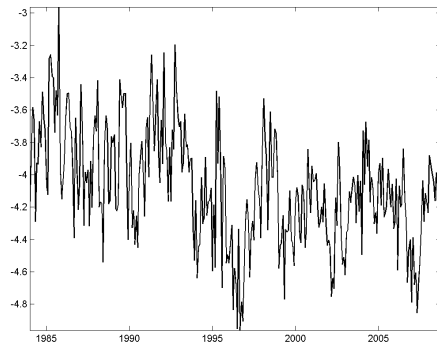
and depends on a comparison of the marginal likelihoods of the drawn model vis-a-vis the previous model of the chain as well as a comparison of the model priors (which are equal in our case). If the number of Monte Carlo draws is large (in our case 5 million) the fraction of draws for the different models converges to the posterior model probability. In order to ensure that the starting value of the chain does not affect the results a burn-in period of 500,000 draws is used.

B.3 Bootstrap Procedure for OOS Evaluation

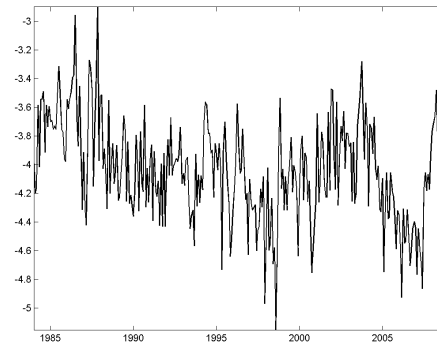
The bootstrap procedure is a model-based wild bootstrap (imposing the null of no predictability by macro-finance variables) and is a variant of the approach considered in [Clark and West \(2006\)](#).²² In each bootstrap iteration the following steps are performed: (i) A series of i.i.d. standard normal innovations η_t is drawn. (ii) AR(1) models are fitted for both the dependent variables $RV_{i;t}$ as well as each of the κ macro-finance variables in z_t and the residuals ($\hat{\epsilon}_t, \hat{\nu}_t$) are saved. (iii) Artificial bootstrap series $RV_{i;t}^{bs}$ and z_t^{bs} are constructed based on the estimated AR(1) parameters and the innovations $\hat{\epsilon}_t\eta_t, \hat{\nu}_t\eta_t$. The starting observations of the bootstrap series $RV_{i;0}^{bs}$ and z_0^{bs} are drawn randomly from the actual series. (iv) The artificial bootstrap data are used to generate recursive forecasts based on models relying on the bootstrapped explanatory macro-finance variables as well as the benchmark AR(1). The corresponding Theil's U statistics TU^b are computed. (v) We compute bootstrap p-values as the fraction of times that Theil's U in the bootstrap samples is below the one observed in-sample. Hence, these p-values are one-sided and test the null of equal predictive performance against the alternative of superior performance of the model including macro-finance predictors vis-a-vis the benchmark. The number of bootstrap iterations is set to 500.

²²The wild bootstrap ensures accurate inference in the presence of conditional heteroskedasticity.

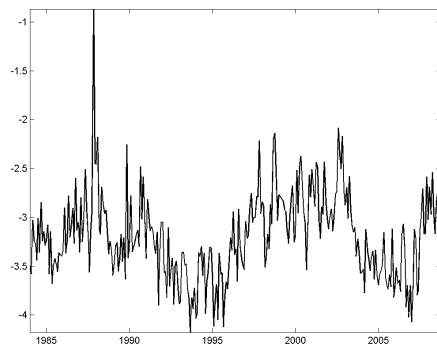
Figure 1. Realized Volatility of Four Asset Classes



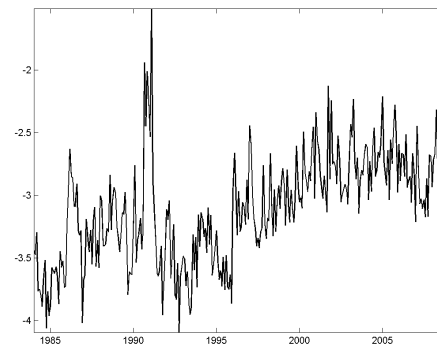
(a) FX



(b) Bonds



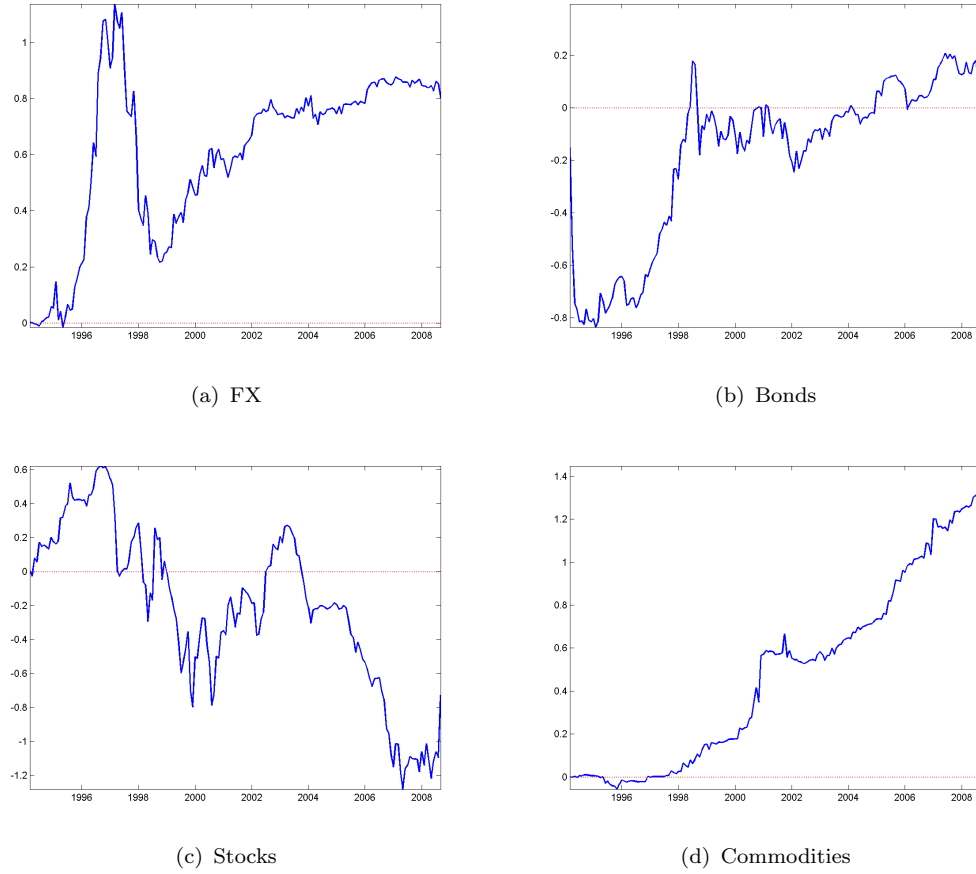
(c) Stocks



(d) Commodities

Notes: The figures show the time series of the log realized volatilities for FX, bond, equity, and commodity markets.

Figure 2. Time-Variation of Out-of-Sample Performance (MC³-BMA)



Notes: The figure shows the time-variation of the out-of-sample performance based on the MC³-BMA approach including predictor variables in levels. Net-SSE is the cumulated difference of squared forecast errors of the benchmark model (AR(1) model) and the model of interest (BMA): $\text{Net-SSE}(\tau) = \sum_{t=1}^{\tau} (e_{uc,t}^2 - e_{c,t}^2)$, where $e_{uc,t}$ is the forecast error of the benchmark, and $e_{c,t}$ is the error of the model of interest. An increase of the slope represents a better forecast performance of the forecast model at the particular point in time.

Table 1. Summary Statistics: Realized Volatility

Panel A.								
	Mean	Std.	Skew.	Kurt.	JB p-val.	AC(1)	AC(2)	AC(3)
FX	-4.076	0.361	0.209	3.000	0.297	0.615	0.499	0.398
Bonds	-4.037	0.357	0.036	3.142	0.500	0.578	0.487	0.477
Stocks	-3.193	0.451	0.845	5.505	0.001	0.650	0.576	0.523
Commod.	-3.105	0.443	0.297	3.132	0.081	0.747	0.706	0.643
Panel B.								
	Correlations					p-values		
	FX	Bonds	Stocks	Commod.	FX	Bonds	Stocks	Commod.
FX	1				—			
Bonds	0.283	1			0.000	—		
Stocks	0.056	0.373	1		0.338	0.000	—	
Commod.	-0.151	0.010	0.196	1	0.010	0.881	0.010	—

Notes: The table shows the summary statistics of the realized volatility for foreign exchange (FX), bond, equity, and commodity markets. The realized volatility is the log of the square root of the realized variance. The reported statistics in Panel A include the mean, standard deviation (Std.), Skewness (Skew.), Kurtosis (Kurt.), the p-value from the Jarque-Bera test for normality (JB p-val.) as well as first (AC(1)), second (AC(2)), and third order (AC(3)) autocorrelation coefficients. Panel B reports the correlations between the four volatility series and the associated p-values.

Table 2. Summary Statistics: Financial and Macroeconomic Predictor Variables

Characteristics of Predictors							Correlation of Predictors and Volatility				
No.	Variable	Abbrev.	Mean	Std.	Skew.	Kurt.	AC(1)	$\rho(z_i, RV_{FX})$	$\rho(z_i, RV_B)$	$\rho(z_i, RV_S)$	$\rho(z_i, RV_C)$
1	Dividend Price Ratio (Log) (*)	DP	-3.78	0.39	0.00	1.84	0.99	0.47	0.32	-0.10	-0.37
2	Book-to-Market Ratio (*)	BM	0.34	0.16	0.99	3.55	0.98	0.51	0.35	-0.07	-0.32
3	Net Equity Expansion (*)	NTIS	0.01	0.02	-0.47	2.95	0.99	-0.19	-0.20	-0.06	-0.19
4	Cross-Sectional Premium (*)	CORPR	0.01	0.02	-0.42	4.77	0.12	0.12	-0.07	0.02	-0.13
5	Pastor-Stambaugh Liquidity Factor	LIQ	-0.02	0.07	-1.81	11.05	0.00	-0.02	-0.21	-0.54	-0.10
6	Return MSCI World	MSCI	0.61	4.03	-0.93	5.57	0.06	-0.05	-0.15	-0.41	-0.13
7	US Market Excess Return	MKTRF	0.54	4.33	-0.91	6.12	0.04	-0.05	-0.14	-0.41	-0.09
8	Size factor	SMB	0.03	3.27	0.87	12.03	-0.03	0.00	0.04	-0.21	0.00
9	Value factor	HML	0.39	3.07	0.16	5.92	0.11	0.06	0.01	0.08	0.07
10	Rel. T-Bill rate	RTB	0.00	0.01	-0.23	2.66	0.97	-0.19	-0.26	-0.14	-0.09
11	Rel. Bond rate	RBR	0.00	0.01	-0.28	5.08	0.88	-0.29	-0.02	0.06	0.02
12	Term spread (*)	TS	0.02	0.01	-0.09	1.84	0.97	0.21	0.42	-0.01	-0.14
13	TED Spread	TED	0.72	0.46	1.38	5.90	0.85	0.30	0.36	0.41	-0.07
14	Long Term Bond Return (*)	LTR	0.01	0.03	-0.04	4.05	0.06	0.14	-0.02	0.10	-0.05
15	T-Bill Rate (Level) (*)	TB	0.40	0.18	0.19	2.82	0.95	0.26	0.05	0.02	-0.31
16	Cochrane Piazzesi Factor	CP	-0.12	2.09	0.27	3.09	0.92	0.21	0.20	-0.06	-0.16
17	Default Spread (*)	DEF	0.95	0.28	0.60	2.43	0.97	0.27	0.46	0.18	0.10
18	Return CRB Spot	CRB	0.02	0.10	-0.07	2.22	0.95	-0.25	0.02	-0.03	0.11
19	Return on Dollar Risk Factor	DOL	0.18	2.03	-0.29	3.64	0.12	-0.06	0.02	0.04	0.04
20	Carry Trade Factor	CT	0.62	2.85	-1.05	4.85	0.15	-0.25	-0.13	-0.17	0.04
21	FX Average Bid-ask spread	BAS	0.12	0.04	2.28	9.75	0.86	0.14	0.01	0.10	-0.12
22	Inflation Rate (*)	INF	0.00	0.00	-0.12	4.27	0.39	0.05	0.04	0.02	0.09
23	Industrial Production Growth	IPGR	0.03	0.03	-0.17	3.81	0.97	-0.06	-0.09	0.00	-0.33
24	Orders	ORD	0.02	0.04	-0.31	4.65	0.83	-0.03	0.00	-0.12	-0.38
25	Average Forward Discount	AFD	0.00	0.00	0.29	2.41	0.86	0.28	0.26	-0.07	-0.15
26	M1 Growth	M1	0.04	0.05	0.39	2.46	0.98	0.37	0.41	-0.06	-0.22
27	Consumer Sentiment	SENT	-0.12	6.08	0.18	3.51	-0.36	0.06	0.06	-0.06	-0.06
28	Purchasing Manager Index	PMI	-0.09	1.91	-0.20	5.07	-0.04	-0.16	0.00	-0.07	-0.07
29	Housing Starts	HS	0.00	0.15	0.13	4.46	-0.49	0.02	-0.03	-0.04	-0.13

Notes: The table shows the summary statistics for the explanatory variables. The reported statistics include the mean, standard deviation (Std.), Skewness (Skew.), Kurtosis (Kurt.), as well as the first order autocorrelation coefficient (AC(1)). In addition, the table reports the correlation coefficient between the contemporaneous value of the predictor z_i and the various realized volatilities. An asterisk (*) denotes that the variable is drawn from the [Goyal and Welch \(2008\)](#) dataset.

Table 3. Predictive Regressions for Financial Volatility, Bayesian Model Averaging

Panel A: FX						Top 5 Models				
No.	Variable	πy	Composite Model			(i)	(ii)	(iii)	(iv)	(v)
			Post. Mean	Post. STD	t-ratio					
1	RV(t-1)	1.000	0.443	0.065	6.814	1	1	1	1	1
2	BM	0.705	0.056	0.044	1.289	1	1	1	1	1
3	AFD	0.551	0.030	0.032	0.939	0	1	0	1	1
4	TED	0.434	0.025	0.033	0.756	0	0	0	0	1
5	CP	0.335	0.015	0.025	0.610	0	0	0	0	0
6	INF	0.302	-0.011	0.020	-0.566	0	0	1	1	0
7	RBR	0.264	-0.012	0.023	-0.510	0	0	0	0	0
8	M1	0.161	0.008	0.021	0.363	0	0	0	0	0
9	DP	0.122	0.004	0.023	0.173	0	0	0	0	0
10	BAS	0.118	0.003	0.012	0.295	0	0	0	0	0

Panel B: Bonds						Top 5 Models				
No.	Variable	πy	Composite Model			(i)	(ii)	(iii)	(iv)	(v)
			Post. Mean	Post. STD	t-ratio					
1	RV(t-1)	1.000	0.363	0.070	5.191	1	1	1	1	1
2	TS	0.925	0.086	0.041	2.073	1	1	1	1	1
3	DEF	0.646	0.046	0.040	1.158	1	1	0	0	1
4	NTIS	0.412	-0.027	0.037	-0.732	0	0	1	1	1
5	TED	0.362	0.019	0.029	0.646	0	1	1	0	0
6	M1	0.117	0.005	0.016	0.295	0	0	0	0	0
7	CT	0.117	0.003	0.012	0.296	0	0	0	0	0
8	INF	0.102	-0.003	0.010	-0.269	0	0	0	0	0
9	TB	0.095	0.003	0.013	0.240	0	0	0	0	0
10	BAS	0.076	0.002	0.008	0.215	0	0	0	0	0

Panel C: Stocks						Top 5 Models				
No.	Variable	πy	Composite Model			(i)	(ii)	(iii)	(iv)	(v)
			Post. Mean	Post. STD	t-ratio					
1	RV(t-1)	1.000	0.434	0.068	6.386	1	1	1	1	1
2	DEF	0.997	0.162	0.041	3.960	1	1	1	1	1
3	BM	0.940	-0.183	0.067	-2.737	1	1	1	1	1
4	TB	0.774	0.073	0.049	1.483	1	1	1	1	0
5	MKTRF	0.555	-0.033	0.035	-0.958	1	0	0	1	0
6	TED	0.241	0.016	0.032	0.492	0	0	0	0	1
7	IPGR	0.172	0.008	0.022	0.381	0	0	0	1	0
8	MSCI	0.132	-0.005	0.018	-0.250	0	0	1	0	0
9	CT	0.107	0.004	0.013	0.283	0	0	0	0	0
10	DP	0.098	-0.008	0.032	-0.264	0	0	0	0	0

Table 3. Continued.

Panel D: Commod.			Composite Model			Top 5 Models				
No.	Variable	πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)	1.000	0.544	0.059	9.181	1	1	1	1	1
2	TB	0.863	-0.118	0.059	-2.007	1	1	1	1	1
3	TS	0.810	-0.095	0.056	-1.712	1	1	1	1	1
4	DEF	0.726	0.051	0.038	1.359	1	1	1	0	1
5	DP	0.238	-0.019	0.040	-0.491	0	0	0	0	0
6	NTIS	0.213	-0.011	0.025	-0.451	0	0	0	1	0
7	ORD	0.133	-0.005	0.015	-0.326	0	0	0	0	1
8	SMB	0.116	0.003	0.011	0.298	0	0	1	0	0
9	M1	0.107	-0.004	0.015	-0.279	0	1	0	0	0
10	RBR	0.082	0.002	0.009	0.226	0	0	0	0	0

Notes: This table reports in-sample results from a Bayesian Model Averaging approach based on an MC^3 algorithm. The lagged dependent variable RV(t-1) is controlled for. The results display the results for the best 10 predictors, as sorted according to the posterior probability of inclusion $\pi|D$ (sorted in descending order). Moreover the table reports the posterior means, standard deviation and t-ratios of the best predictors (reflecting model uncertainty). Inclusion of the specific variable in the Top 5 models (according to the posterior model probability) is indicated by 1.

Table 4. Predictive Regressions for Financial Volatility: Classical Model Selection

Panel A: FX					Panel B: Bonds					Panel C: Stocks					Panel D: Commodities				
Top Models					Top Models					Top Models					Top Models				
Bench.					Bench.					Bench.					Bench.				
(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)
BM	0.095 [0.000]	0.089 [0.000]	0.081 [0.000]	0.092 [0.000]	0.062 [0.000]	0.064 [0.000]	0.084 [0.000]	0.017 [0.000]	0.021 [0.000]	NTIS	0.381 [0.000]	0.34 [0.000]	0.33 [0.000]	0.355 [0.000]	0.378 [0.000]	0.571 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
RTB	0.017 [0.000]	0.017 [0.000]	0.017 [0.000]	0.017 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	TS	0.064 [0.000]	0.084 [0.000]	0.015 [0.000]	0.015 [0.000]	0.018 [0.000]	0.018 [0.000]	0.018 [0.000]	0.018 [0.000]	0.018 [0.000]
TED	0.017 [0.000]	0.017 [0.000]	0.017 [0.000]	0.017 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	TED	0.017 [0.000]	0.019 [0.000]	0.021 [0.000]	0.023 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]
CP	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	0.042 [0.000]	DEF	0.082 [0.000]	0.065 [0.000]	0.055 [0.000]	0.055 [0.000]	0.055 [0.000]	0.055 [0.000]	0.055 [0.000]	0.055 [0.000]	0.055 [0.000]
INF	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	-0.039 [0.000]	RV[t-1]	0.381 [0.000]	0.34 [0.000]	0.33 [0.000]	0.355 [0.000]	0.378 [0.000]	0.571 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
adj. R^2	0.412 [0.000]	0.402 [0.000]	0.418 [0.000]	0.409 [0.000]	0.426 [0.000]	0.426 [0.000]	0.426 [0.000]	0.426 [0.000]	0.426 [0.000]	adj. R^2	0.388 [0.000]	0.396 [0.000]	0.396 [0.000]	0.395 [0.000]	0.384 [0.000]	0.317 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
BIC	-2.533 [0.000]	-2.532 [0.000]	-2.528 [0.000]	-2.528 [0.000]	-2.526 [0.000]	-2.526 [0.000]	-2.526 [0.000]	-2.526 [0.000]	-2.526 [0.000]	BIC	-2.495 [0.000]	-2.493 [0.000]	-2.492 [0.000]	-2.491 [0.000]	-2.49 [0.000]	-2.418 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
Panel C: Stocks					Panel D: Commodities					Panel E: Bonds					Panel F: Commodities				
Top Models					Top Models					Top Models					Top Models				
Bench.					Bench.					Bench.					Bench.				
(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)
DP	-0.092 [0.000]	-0.092 [0.000]	-0.092 [0.000]	-0.092 [0.000]	-0.029 [0.000]	-0.029 [0.000]	-0.029 [0.000]	-0.029 [0.000]	-0.029 [0.000]	DP	0.381 [0.000]	0.34 [0.000]	0.33 [0.000]	0.355 [0.000]	0.378 [0.000]	0.571 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
BM	-0.113 [0.000]	-0.113 [0.000]	-0.113 [0.000]	-0.113 [0.000]	-0.084 [0.000]	-0.084 [0.000]	-0.084 [0.000]	-0.084 [0.000]	-0.084 [0.000]	SMB	0.064 [0.000]	0.084 [0.000]	0.015 [0.000]	0.015 [0.000]	0.018 [0.000]	0.018 [0.000]	0.018 [0.000]	0.018 [0.000]	0.018 [0.000]
DEF	0.124 [0.000]	0.124 [0.000]	0.124 [0.000]	0.124 [0.000]	0.119 [0.000]	0.119 [0.000]	0.119 [0.000]	0.119 [0.000]	0.119 [0.000]	TS	-0.087 [0.000]	-0.08 [0.000]	-0.08 [0.000]	-0.08 [0.000]	-0.08 [0.000]	-0.08 [0.000]	-0.08 [0.000]	-0.08 [0.000]	-0.08 [0.000]
CT	0.028 [0.000]	0.028 [0.000]	0.028 [0.000]	0.028 [0.000]	0.031 [0.000]	0.031 [0.000]	0.031 [0.000]	0.031 [0.000]	0.031 [0.000]	TB	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]	0.019 [0.000]
AFD	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	0.022 [0.000]	ORD	0.027 [0.000]	0.026 [0.000]	0.027 [0.000]	0.027 [0.000]	0.027 [0.000]	0.027 [0.000]	0.027 [0.000]	0.027 [0.000]	0.027 [0.000]
RV[t-1]	0.526 [0.000]	0.534 [0.000]	0.517 [0.000]	0.533 [0.000]	0.525 [0.000]	0.525 [0.000]	0.525 [0.000]	0.525 [0.000]	0.525 [0.000]	RV[t-1]	0.598 [0.000]	0.573 [0.000]	0.6 [0.000]	0.599 [0.000]	0.63 [0.000]	0.735 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
adj. R^2	0.418 [0.000]	0.415 [0.000]	0.421 [0.000]	0.418 [0.000]	0.417 [0.000]	0.417 [0.000]	0.417 [0.000]	0.417 [0.000]	0.417 [0.000]	adj. R^2	0.566 [0.000]	0.571 [0.000]	0.569 [0.000]	0.562 [0.000]	0.554 [0.000]	0.527 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]
BIC	-2.125 [0.000]	-2.118 [0.000]	-2.112 [0.000]	-2.108 [0.000]	-2.107 [0.000]	-2.107 [0.000]	-2.107 [0.000]	-2.107 [0.000]	-2.107 [0.000]	BIC	-2.43 [0.000]	-2.428 [0.000]	-2.423 [0.000]	-2.422 [0.000]	-2.42 [0.000]	-2.378 [0.000]	0.019 [0.000]	0.015 [0.000]	0.018 [0.000]

Notes: The table shows the results from the predictive regressions for the five top-performing models based upon the BIC as well as for the AR(1) benchmark model where the explained variable is realized volatility for different asset classes. The lagged dependent variable $RV(t-1)$ is controlled for. Robust standard errors are reported in brackets while bootstrap p-values (obtained by a parametric bootstrap) are displayed in parentheses.

Table 5. Out-of-Sample Forecast Evaluation

Panel A: FX				Panel B: Bonds			
	Top	BMA MC3	EW Top10		Top	BMA MC3	EW Top10
TU	0.980	0.971	0.969	TU	0.991	0.987	0.990
$\#TU^{bs} < TU$	0.002	0.000	0.000	$\#TU^{bs} < TU$	0.006	0.004	0.008
R_{OOS}^2	0.039	0.058	0.061	R_{OOS}^2	0.018	0.026	0.020
MZ GLS	7.974	5.944	5.050	MZ GLS	0.040	0.253	0.221
p-val.	0.019	0.051	0.080	p-val.	0.981	0.881	0.895
Panel C: Stocks				Panel D: Commodities			
	Top	BMA MC3	EW Top10		Top	BMA MC3	EW Top10
TU	1.028	1.020	1.015	TU	0.961	0.949	0.949
$\#TU^{bs} < TU$	0.873	0.870	0.808	$\#TU^{bs} < TU$	0.000	0.000	0.000
R_{OOS}^2	-0.057	-0.040	-0.030	R_{OOS}^2	0.077	0.100	0.100
MZ GLS	8.167	7.710	7.904	MZ GLS	22.786	21.384	20.138
p-val.	0.017	0.021	0.019	p-val.	0.000	0.000	0.000

Notes: The table shows the results from the evaluation of out-of-sample forecasts based on the MC³ sampling algorithm. TOP denotes the forecast based on the model, which takes the highest posterior model probability when the forecast is made. BMA is the forecast obtained by combining forecasts according to Bayesian Model Averaging, whereas EW Top10 is the equally weighted forecast of the 10 best models (in terms of posterior model probability) at the time of the forecast. The reported statistics include Theil's U which is the ratio of the RMSE of the model of interest and the RMSE of the benchmark model (TU), the out-of-sample R^2 of [Campbell and Thompson \(2008\)](#). $\#TU^{bs} < TU$ denotes the bootstrap p-value for testing equal predictive performance of the macro-finance augmented model and the AR(1) benchmark against the alternative of superior performance of the model including macro-finance predictors. The bootstrap procedure follows a model-based wild bootstrap methodology as described in section 3. MZ GLS denotes the GLS version of the Mincer-Zarnowitz statistic.

Table 6. Predictive Regressions for Financial Volatility: BMA, Incl. Squared Terms of Predictive Variables

Panel A: FX			Composite Model				Top 5 Models				
No.	Variable		πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)		1.000	0.449	0.061	7.416	1	1	1	1	1
2	BM	Lv	0.673	0.056	0.044	1.268	1	1	1	1	1
3	AFD	Lv	0.528	0.032	0.035	0.919	0	1	1	0	0
4	TED	Lv	0.289	0.017	0.029	0.574	0	0	0	0	0
5	BAS	Sq	0.282	0.004	0.007	0.561	0	0	1	0	1
6	INF	Lv	0.195	-0.007	0.017	-0.439	0	0	0	1	0
7	CP	Lv	0.171	0.007	0.019	0.398	0	0	0	0	0
8	RBR	Lv	0.144	-0.006	0.018	-0.353	0	0	0	0	0
9	DP	Lv	0.115	0.006	0.023	0.272	0	0	0	0	0
10	DEF	Sq	0.095	0.003	0.011	0.279	0	0	0	0	0
Panel B: Bonds			Composite Model				Top 5 Models				
No.	Variable		πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RBR	Sq	1.000	0.043	0.009	4.818	1	1	1	1	1
2	RV(t-1)		0.997	0.282	0.067	4.238	1	1	1	1	1
3	TS	Lv	0.993	0.119	0.029	4.091	1	1	1	1	1
4	NTIS	Lv	0.743	-0.055	0.037	-1.466	1	1	1	0	0
5	CT	Sq	0.693	-0.018	0.014	-1.287	1	0	1	1	1
6	TED	Lv	0.353	0.021	0.032	0.656	0	0	1	1	1
7	SENT	Sq	0.118	-0.002	0.008	-0.320	0	0	0	0	0
8	TB	Sq	0.111	0.003	0.010	0.305	0	0	0	0	0
9	HML	Sq	0.108	0.002	0.005	0.303	0	0	0	0	0
10	DEF	Lv	0.102	0.005	0.017	0.293	0	0	0	0	0
Panel C: Stocks			Composite Model				Top 5 Models				
No.	Variable		πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)		1.000	0.416	0.074	5.638	1	1	1	1	1
2	DEF	Lv	0.991	0.150	0.040	3.729	1	1	1	1	1
3	BM	Lv	0.896	-0.161	0.068	-2.359	1	1	1	1	1
4	TED	Lv	0.637	0.072	0.063	1.144	1	1	0	0	0
5	RTB	Sq	0.602	-0.028	0.026	-1.079	1	1	0	0	1
6	MKTRF	Lv	0.588	-0.040	0.038	-1.054	1	1	0	1	1
7	TED	Sq	0.403	-0.016	0.021	-0.745	1	1	0	0	0
8	TB	Lv	0.314	0.027	0.044	0.612	0	0	1	1	1
9	SMB	Sq	0.176	0.002	0.006	0.413	0	1	0	0	0
10	HML	Sq	0.174	0.004	0.009	0.409	0	0	0	0	0

Table 6. Continued.

Panel D: Commod.			Composite Model				Top 5 Models				
No.	Variable		πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)		1.000	0.538	0.061	8.831	1	1	1	1	1
2	TB	Lv	0.672	-0.093	0.072	-1.285	1	1	1	0	0
3	TS	Lv	0.638	-0.075	0.063	-1.194	1	1	1	0	0
4	DEF	Lv	0.565	0.039	0.039	1.009	1	1	1	0	0
5	DP	Lv	0.358	-0.034	0.049	-0.685	0	0	0	1	1
6	AFD	Sq	0.286	-0.011	0.020	-0.569	0	0	1	1	0
7	INF	Sq	0.204	0.005	0.011	0.453	0	0	0	1	0
8	NTIS	Lv	0.173	-0.009	0.022	-0.408	0	0	0	0	1
9	M1	Sq	0.114	-0.003	0.011	-0.314	0	1	0	0	0
10	TB	Sq	0.111	0.003	0.011	0.307	0	0	0	0	0

Notes: This table reports in-sample results from a Bayesian Model Averaging approach based on an MC^3 algorithm. Predictive variables are included in levels (Lv) and squared terms (Sq). The lagged dependent variable RV(t-1) is controlled for. The results display the results for the best 10 predictors, as sorted according to the posterior probability of inclusion $\pi|D$ (sorted in descending order). Moreover the table reports the posterior means, standard deviation and t-ratios of the best predictors (reflecting model uncertainty). Inclusion of the specific variable in the Top 5 models (according to the posterior model probability) is indicated by 1.

Table 7. Out-of-Sample Forecast Evaluation: Non-Linearities and Predictors in Absolute Values

Panel A: Levels and Squared Terms of Predictors							
<i>A.1.: FX</i>	Top	BMA MC3	EW Top10	<i>A.2.: Bonds</i>	Top	BMA MC3	EW Top10
TU	0.988	0.973	0.974	TU	1.008	0.981	0.978
#TU ^{bs} <TU	0.010	0.000	0.000	#TU ^{bs} <TU	0.378	0.002	0.000
R^2_{OOS}	0.024	0.053	0.051	R^2_{OOS}	-0.016	0.038	0.044
MZ GLS	14.878	10.432	9.772	MZ GLS	0.806	0.052	0.240
p-val.	0.001	0.005	0.008	p-val.	0.668	0.975	0.887
<i>A.3.: Stocks</i>	Top	BMA MC3	EW Top10	<i>A.4.: Commod.</i>	Top	BMA MC3	EW Top10
TU	1.088	1.058	1.049	TU	0.970	0.964	0.960
#TU ^{bs} <TU	0.892	0.878	0.844	#TU ^{bs} <TU	0.000	0.000	0.000
R^2_{OOS}	-0.184	-0.119	-0.100	R^2_{OOS}	0.059	0.072	0.079
MZ GLS	13.556	8.937	7.803	MZ GLS	24.412	24.250	23.230
p-val.	0.001	0.012	0.020	p-val.	0.000	0.000	0.000
Panel B: Absolute Values of Predictors							
<i>B.1.: FX</i>	Top	BMA MC3	EW Top10	<i>B.2.: Bonds</i>	Top	BMA MC3	EW Top10
TU	1.002	1.001	1.001	TU	1.015	1.012	1.011
#TU ^{bs} <TU	0.358	0.188	0.176	#TU ^{bs} <TU	0.720	0.712	0.698
R^2_{OOS}	-0.003	-0.003	-0.002	R^2_{OOS}	-0.029	-0.025	-0.022
MZ GLS	28.971	27.492	27.186	MZ GLS	3.322	2.872	2.741
p-val.	0.000	0.000	0.000	p-val.	0.190	0.238	0.254
<i>B.3.: Stocks</i>	Top	BMA MC3	EW Top10	<i>B.4.: Commod.</i>	Top	BMA MC3	EW Top10
TU	1.080	1.075	1.071	TU	1.012	1.009	1.006
#TU ^{bs} <TU	0.976	0.980	0.980	#TU ^{bs} <TU	0.700	0.650	0.506
R^2_{OOS}	-0.167	-0.155	-0.148	R^2_{OOS}	-0.024	-0.019	-0.013
MZ GLS	5.678	9.959	10.164	MZ GLS	16.733	18.874	18.852
p-val.	0.059	0.007	0.006	p-val.	0.000	0.000	0.000

Notes: The table shows the results from the evaluation of out-of-sample forecasts based on the MC^3 approach when using levels and squared terms of predictors (Panel A) or using levels and absolute values of the predictors (Panel B). TOP denotes the forecast based on the model, which takes the highest posterior model probability when the forecast is made. BMA is the forecast obtained by combining forecasts according to Bayesian Model Averaging, whereas EW Top10 is the equally weighted average of the forecasts of the 10 best models (in terms of posterior model probability) at the time of the forecast. The reported statistics include the ratio of the RMSE of the model of interest and the RMSE of the benchmark model (Theil's U, denoted TU), the OOS R^2 of Campbell and Thompson (2008). #TU^{bs}<TU denotes the bootstrap p-value of a test for equal predictive performance of the macro-finance augmented model and the AR(1) benchmark against the alternative of superior performance of the model including macro-finance predictors. MZ GLS denotes the GLS version of the Mincer-Zarnowitz statistic.

Table 8. Predictive Regressions for Financial Volatility: BMA,
Absolute Values of Predictors

Panel A: FX			Composite Model			Top 5 Models				
No.	Variable	πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)	1.000	0.488	0.063	7.766	1	1	1	1	1
2	DOL	0.973	0.103	0.035	2.916	1	1	1	1	1
3	CP	0.301	0.020	0.035	0.566	0	1	0	0	0
4	TB	0.190	0.011	0.026	0.410	0	0	1	0	0
5	AFD	0.157	0.009	0.026	0.360	0	0	0	0	0
6	RTB	0.144	0.007	0.021	0.340	0	0	0	0	1
7	SMB	0.139	-0.006	0.017	-0.332	0	0	0	0	0
8	DEF	0.127	0.007	0.021	0.312	0	0	0	1	0
9	RBR	0.106	0.004	0.015	0.275	0	0	0	0	0
10	BAS	0.101	0.004	0.014	0.266	0	0	0	0	0

Panel B: Bonds			Composite Model			Top 5 Models				
No.	Variable	πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)	1.000	0.408	0.062	6.533	1	1	1	1	1
2	RBR	0.999	0.130	0.030	4.319	1	1	1	1	1
3	TB	0.939	0.097	0.040	2.442	1	1	1	1	1
4	CT	0.799	-0.061	0.039	-1.558	1	1	1	0	0
5	ORD	0.294	0.017	0.030	0.556	0	1	0	0	1
6	NTIS	0.274	0.017	0.032	0.528	0	0	1	0	0
7	TS	0.139	-0.008	0.025	-0.332	0	0	0	0	0
8	SENT	0.135	-0.006	0.019	-0.326	0	0	0	0	0
9	TED	0.115	0.006	0.020	0.291	0	0	0	0	0
10	DOL	0.090	0.003	0.014	0.245	0	0	0	0	0

Panel C: Stocks			Composite Model			Top 5 Models				
No.	Variable	πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)	1.000	0.599	0.058	10.401	1	1	1	1	1
2	IPGR	0.313	0.025	0.043	0.591	0	1	0	1	0
3	DEF	0.214	0.017	0.037	0.452	0	0	1	0	0
4	RTB	0.200	-0.014	0.032	-0.430	0	0	0	1	0
5	MKTRF	0.104	0.006	0.022	0.277	0	0	0	0	0
6	ORD	0.092	0.005	0.018	0.252	0	0	0	0	0
7	MSCI	0.092	0.005	0.019	0.253	0	0	0	0	0
8	HML	0.086	0.004	0.015	0.243	0	0	0	0	0
9	DP	0.083	0.006	0.024	0.233	0	0	0	0	0
10	LIQ	0.083	-0.004	0.015	-0.236	0	0	0	0	0

Table 8. Continued.

Panel D: Commod.			Composite Model			Top 5 Models				
No.	Variable	πy	Post. Mean	Post. STD	t-ratio	(i)	(ii)	(iii)	(iv)	(v)
1	RV(t-1)	1.000	0.638	0.057	11.239	1	1	1	1	1
2	INF	0.919	0.090	0.039	2.286	1	1	1	1	1
3	AFD	0.499	-0.044	0.051	-0.866	1	0	0	1	0
4	CP	0.381	-0.028	0.041	-0.685	0	1	0	1	0
5	M1	0.143	-0.009	0.025	-0.342	0	0	0	0	1
6	SMB	0.126	0.005	0.016	0.316	0	0	0	0	0
7	IPGR	0.114	-0.005	0.017	-0.296	0	0	0	0	0
8	HS	0.073	-0.003	0.012	-0.214	0	0	0	0	0
9	RBR	0.068	-0.002	0.011	-0.203	0	0	0	0	0
10	RTB	0.064	-0.002	0.012	-0.190	0	0	0	0	0

Notes: This table reports in-sample results from a Bayesian Model Averaging approach based on an MC^3 algorithm. Predictive variables are included in absolute values. The lagged dependent variable RV(t-1) is controlled for. The results display the results for the best 10 predictors, as sorted according to the posterior probability of inclusion $\pi|D$ (sorted in descending order). Moreover the table reports the posterior means, standard deviation and t-ratios of the best predictors (reflecting model uncertainty). Inclusion of the specific variable in the Top 5 models (according to the posterior model probability) is indicated by 1.

Table A.1. Predictive Variables: Data Sources and Construction

No.	Variable	Abbrev.	Data Source	Description
1	Dividend Price Ratio (Log) (*)	DP	Goyal-Welch Data	Dividends over the past year (12-month moving sum) relative to current market prices (in logs); S&P 500 index.
2	Book-to-Market Ratio (*)	BM	Goyal-Welch Data	Ratio of book value to market value for the Dow Jones Industrial Average.
3	Net Equity Expansion (*)	NTIS	Goyal-Welch Data	Ratio of 12-month moving sums of net issues by NYSE listed stocks divided by total NYSE market capitalization (end-of-year).
4	Cross-Sectional Premium (*)	CROPR	Goyal-Welch Data	Measure of the relative valuations of high- and low-beta stocks by Polk, Thompson, and Vuolteenaho (2006).
5	Pastor-Stambaugh Liquidity Factor (†)	LIQ	CRSP	Measure of stock market liquidity based on price reversals.
6	Return MSCI World	MSCI	Datastream	Return on the MSCI world stock market index.
7	US Market Excess Return	MKTRF	K. French	Fama-French's market factor: U.S. stock market return minus one-month T-Bill rate.
8	Size factor	SMB	K. French	Fama-French's SMB factor: Return on small stocks minus return on big stocks.
9	Value factor	HML	K. French	Fama-French's HML factor: Return on value stocks minus return on growth stocks.
10	Rel. T-Bill rate	RTB	Goyal-Welch Data	T-Bill rate minus its 12 month moving average.
11	Rel. Bond rate	RBR	Goyal-Welch Data	Long-term bond yield minus its 12 month moving average.
12	Term spread (*)	TS	Goyal-Welch Data	Difference of long-term bond yield and three-month T-Bill rate.
13	TED Spread	TED	Datastream	Measure of illiquidity, difference of Eurodollar rate and T-Bill rate.
14	Long Term Bond Return (*)	LTR	Goyal-Welch Data	Rate of return on long term government bonds.
15	T-Bill Rate (Level) (*)	TB	Goyal-Welch Data	Three-month T-Bill rate.
16	Cochrane Piazzesi Factor (‡)	CP	CRSP	Measure of bond risk premia; recursively estimated based on Fama-Bliss file in CRSP.
17	Default Spread (*)	DEF	Goyal-Welch Data	Measure of default risk of corporate bonds; difference of BAA bond yields and AAA bond yields.
18	Return CRB Spot	CRB	Datastream	Measure of growth in commodity prices; annual log difference of CRB spot index.
19	Return on Dollar Risk Factor (¶)	DOL	BBJ/Reuters (Datstream)	FX risk premium measure; Average premium for bearing FX risk.
20	Carry Trade Factor (¶)	CT	BBJ/Reuters (Datstream)	Return on high interest rate currencies minus return on low interest rate currencies.
21	FX Average Bid-ask spread (††)	BAS	BBJ/Reuters (Datstream)	Measure of illiquidity in the foreign exchange market calculated from bid-ask spreads.
22	Inflation Rate (*)	INF	Datastream	Year-over year (log) growth rate of the U.S. consumer price index.
23	Industrial Production Growth	IPGR	Datastream	Year-over year (log) growth rate of U.S. industrial production.

Table A.1. Continued.

No.	Variable	Abbrev.	Data Source	Description
24	Orders	ORD	Datastream	New orders of consumer goods and materials; year-over year (log) growth rate.
25	Average Forward Discount (‡)	AFD	BBI/Reuters (Datastream)	Aggregate predictor of FX returns calculated from forward rates and spot rates.
26	M1 Growth	M1	Datastream	Year-over-year (log) growth rate of U.S. M1.
27	Consumer Sentiment	SENT	Datastream	Monthly change in consumer sentiment.
28	Purchasing Manager Index	PMI	Datastream	Monthly change in purchasing manager index.
29	Housing Starts	HS	Datastream	Monthly change in housing started.

Notes:

(*) Variable is among the predictors considered in [Goyal and Welch \(2008\)](#). See [Goyal and Welch \(2008\)](#) for further details on data construction.

(†) See [Pastor and Stambaugh \(2003\)](#) for further details on data construction.

(‡) See [Cochrane and Piazzesi \(2005\)](#) for further details on data construction.

(¶) See [Lustig, Roussanov, and Verdelhan \(2010a\)](#) for further details on data construction.

(††) See [Menkhoff, Sarno, Schmeling, and Schrimpf \(2010\)](#) for further details on data construction.

(‡‡) See [Lustig, Roussanov, and Verdelhan \(2010b\)](#) for further details on data construction.