The Swaption Cube

Abstract

We use a comprehensive database of inter-dealer quotes to conduct the first empirical analysis of the swaption cube. Using a model independent approach, we establish a set of stylized facts regarding the cross-sectional and time-series variation in conditional volatility and skewness of the swap rate distributions implied by the swaption cube. We then develop and estimate a dynamic term structure model that is consistent with these stylized facts and use it to infer volatility and skewness of the risk-neutral and physical swap rate distributions. Finally, we investigate the fundamental drivers of these distributions. In particular, we find that volatility, volatility risk premia, skewness, and skewness risk premia are significantly related to the characteristics of agents' belief distributions for the macro-economy, with GDP beliefs being the most important factor in the USD market and inflation beliefs being the most important factor in the EUR market. This is consistent with differences in monetary policy objectives in the two markets.

1 Introduction

In this paper, we conduct an extensive empirical analysis of the market for interest rate swaptions – options to enter into interest rate swaps – using a novel data set. Understanding the pricing of swaptions is important for several reasons. By some measures (such as the notional amount of outstanding contracts) the swaption market is the world's largest options market. Furthermore, many standard fixed income securities, such as fixed rate mortgage-backed securities and callable agency securities, imbed swaption-like options, and swaptions are used to hedge and risk-manage these securities in addition to many exotic interest rate derivatives and structured products. Moreover, many large corporations are active in the swaption market either directly or indirectly (through the issuance and swapping of callable debt), and a better understanding of the pricing of swaptions may, therefore, have implications for corporate finance decisions.[1]

The importance of the swaption market has spurred a number of empirical studies over the past decade.[2] Our paper differs from these in two important ways. First, all existing studies are limited to only using data on at-the-money (ATM) swaptions. In contrast, we analyze a proprietary data set with extensive information about non-ATM swaptions, which sheds new light on the swaption market. Second, existing studies are mostly concerned with the pricing and hedging of swaptions using reduced-form models. Although we also utilize a reduced-form dynamic term structure model, a key objective of the paper is to understand the fundamental drivers of prices and risk premia in the swaption market.

The paper takes advantage of a unique data set from the largest inter-dealer broker in the interest rate derivatives market, which records prices of swaptions along three dimensions: the maturities of the underlying swaps (the swap tenors), the expiries of the options, and the option strikes. This three-dimensional grid of prices is known, among market participants, as the *swaption cube*. The range along all three dimensions is wide with swap tenors from 2 years to 30 years, option expiries from 1 month to 10 years, and strike intervals up to 800 basis points. The data covers more than eight years from 2001 to 2010, and spans two recessions and

[1] A 2009 survey by the International Swaps and Derivatives Association found that 88.3 percent of the Fortune Global 500 companies use interest rate derivatives, such as swaps and swaptions, to manage interest rate risk.

[2] See Longstaff, Santa-Clara, and Schwartz (2001a), Longstaff, Santa-Clara, and Schwartz (2001b), Driessen, Klaassen, and Melenberg (2003), Fan, Gupta, and Ritchken (2003), de Jong, Driessen, and Pelsser (2004), Han (2007), Joslin (2007), Duarte (2008), Trolle and Schwartz (2009), and Carr, Garbaix, and Wu (2009).

the financial crisis. Moreover, the data contains both USD and EUR denominated swaptions – by far the most liquid currencies – allowing us to ascertain the robustness of our results across different markets.

We first analyze the swaption cube from a model independent perspective. For a given swap maturity and option expiry, we compute conditional moments (under the appropriate pricing measure[3]) of the swap rate distribution at a time horizon equal to the option expiry. This is done by suitably integrating over swaptions with different strikes. For instance, supposing we consider the 1-year option on the 10-year swap, then the strike-dimension of swaption prices gives us conditional moments of the 10-year swap rate distribution in 1-year's time. Since we observe options with a wide range of expiries, for each swap maturity we obtain a term structure of conditional swap rate moments.

We investigate how conditional moments vary with option expiry, swap maturity, and across time. Results regarding conditional volatility are largely consistent with existing studies using ATM implied swaption volatilities, although we here compute conditional volatility by taking the entire implied volatility smile into account. It is for the higher-order moments that we obtain a series of new stylized facts regarding the cross-sectional and time-series variation. Conditional skewness is mostly positive on average. At a given option expiry, conditional skewness, on average, decreases with swap maturity and is negative in some cases. For a given swap maturity, conditional skewness, on average, increases with option expiry. Furthermore, conditional skewness exhibits significant variation over time to the extent that the sign of conditional skewness sometimes changes. However, changes in conditional skewness are largely unrelated to changes in swap rates and volatility and show strong common variation across swap maturities and option expiries, strongly indicating that skewness risk is largely orthogonal to term structure and volatility risks. Conditional swap rate distributions always have fat tails. However, conditional kurtosis exhibits less systematic variation than is the case for conditional skewness. For this reason, we mainly focus on conditional volatility and skewness in the paper.

Motivated by the model-independent analysis, we develop a dynamic model of the term structure of swap rates. The model features two volatility factors, which may be partially spanned by the term structure, and swaptions can be priced with a fast and accurate Fourier-based pricing formula. By specifying shocks to the term structure judiciously (a specification that encompasses "level", "slope", and "curvature" shocks), the model reduces to a particular

[3]In the case of swaptions, the appropriate pricing measure is not the risk-neutral measure, but rather the annuity measure, see the discussion in Section 3.

case of an affine term structure model.[4] The model is estimated by maximum-likelihood on a panel data set, which includes all the swaptions and underlying swap rates in the swaption cubes. We show that a parsimonious specification with three term structure factors and two volatility factors, which only differ from each other in their correlations with the term structure factors, is able to capture most of the cross-sectional and time-series variation in conditional volatility and skewness of the swap rate distributions under the pricing measure.

The main purpose of imposing a dynamic term structure model is to infer the conditional swap rate distributions under the risk-neutral measure as well as the physical measure. This allows us to study the pricing of risk in the swaption market. We show that the risk-neutral swap rate distributions on average exhibit higher volatility and are more skewed towards higher rates than the swap rate distributions under the physical measure.

Ultimately, we are interested in understanding the fundamental drivers of the conditional swap rate distributions – in particular the effects of macro-economic uncertainty. To quantify macro-economic uncertainty, we infer agents' perceived probability distributions (which we call agents' *belief distributions*) for 1-year ahead real GDP growth and inflation from the survey of professional forecasters conducted in both the US and the Eurozone.[5] We then regress volatility and skewness of the physical swap rate distributions as well as volatility risk premia (defined as the differences between physical and risk-neutral volatility) and skewness risk premia (defined as the differences between physical and risk-neutral skewness) on the dispersion and skewness of agents' belief distributions for future real GDP growth and inflation. We also control for other factors that may have an effect on swap rate distributions, including volatility and skewness

[4]In addition to facilitating estimation, the affine representation also facilitates pricing of complex interest rate derivatives by simulation. A large class of popular derivatives known as callable LIBOR exotics (including Bermudan swaptions, callable capped floaters, callable range accruals, and target redemption notes) are priced by simulation, often using the LSM scheme of Longstaff and Schwartz (2001) to address early-exercise features. In the work-horse model in the financial industry, the LIBOR market model, simulation is time-consuming since each forward LIBOR rate must be simulated by itself. To price a typical 30-year structure in the USD market, 120 forward LIBOR rates (with complicated drift conditions) must be simulated. In contrast, the model considered in this paper has a limited set of state variables (with simple affine dynamics) making it much faster to obtain prices and hedge-ratios.

[5]Our focus on expectations about future growth and inflation, as opposed to their current values, is consistent with an influential paper by Clarida, Gali, and Gertler (2000), which finds that the Federal Reserve adjusts monetary policy in response to deviations in expected output and inflation from their respective target levels (i.e. it follows a so-called forward-looking Taylor rule).

of the equity index return distribution, market-wide liquidity, and refinancing activity.[6]

In the USD market, we find that dispersion of agents' belief distribution for future real GDP growth has a significantly positive effect on volatility of the physical swap rate distributions and a significantly negative impact on volatility risk premia. This is consistent with equilibrium models, primarily developed for equity derivatives, in which an increase in uncertainty and/or disagreement among agents about fundamentals increases both risk-neutral and physical volatility as well as the wedge between the two.[7] Furthermore, skewness of agents' GDP beliefs has a significantly positive effect on skewness of the physical swap rate distributions. Interestingly, the swap rate distributions are less related to the characteristics of agents' inflation beliefs.

In contrast, in the EUR market, the swap rate distributions are more related to the characteristics of agents' belief distribution for future inflation than for future real GDP growth. For instance, dispersion of agents' inflation beliefs has a significantly positive effect on volatility of the physical swap rate distributions, and a significantly negative impact on volatility risk premia. Furthermore, skewness of agents' inflation beliefs has a significantly positive effect on skewness of the physical swap rate distributions, and a significantly negative impact on skewness risk premia – the latter possibly reflecting agents' dislike for high inflation states. One likely reason for these differences between the two markets is that the primary policy goal of the European Central Bank is to maintain price stability, whereas the Federal Reserve has a dual mandate of maximum employment and price stability, leading it to place relatively more emphasis on expectations for real GDP growth when settings interest rates.

We also find that various dimensions of the swap rate distributions are related to the characteristics of the equity index return distribution as well as market-wide liquidity. Refinancing activity appears to play a more modest role than previous papers have suggested, which may, in part, be due to the Federal Reserve's massive involvement in the MBS market in the latter part of the sample period. Since it does not engage in convexity hedging, this reduces the

[6]In an extensive study, Duarte (2008) finds that refinancing activity has an impact on the pricing of ATM swaptions.

[7]This is, for instance, the case in long run risk models, where agents have preferences for early resolution of uncertainty, and macro-economic uncertainty is stochastic (see, e.g., Eraker and Shaliastovich (2008), Bollerslev, Tauchen, and Zhou (2009), Bollerslev, Sizova, and Tauchen (2009), Drechsler and Yaron (2009), and Shaliastovich (2009)), in models where agents have incomplete information and face fundamentals subject to regime switches (see, e.g., David and Veronesi (2002, 2009)), or in models where agents have incomplete information and heterogeneous beliefs (see, e.g., Buraschi and Jiltsov (2006) and Buraschi, Trojani, and Vedolin (2009)).

effect of refinancing activity on the swaption market.

The literature on interest rate derivatives is vast. A related set of papers analyze volatility smiles in the market for interest rate caps/floors.[8] That market is a subset of the swaption market, since a cap is a basket of caplets, which are options on LIBOR rates of a particular maturity. The swaption cube data set is much more extensive, since the underlying swaps have a wide range of maturities.[9] Compared to those studies that analyze ATM swaptions and those that analyze cap/floor volatility smiles, we establish a series of new stylized facts about swap rate distributions, develop a dynamic term structure model that matches these stylized facts, and provide a detailed analysis of the fundamental drivers of the swap rate distributions.

Our paper is also related to a growing literature linking the term structure of interest rates to macro factors.[10] This literature is mainly based on Gaussian models, and, consequently, primarily concerns itself with the determinants of the conditional mean of interest rates. We complement this literature by studying the determinants of the conditional volatility and skewness of interest rates, which are critical for derivatives prices.[11]

The rest of the paper is organized as follows: Section 2 describes the swaption cube data. Section 3 uses a model independent approach to establish several stylized facts about the swap rate distributions under the pricing measure. Section 4 describes and evaluates a dynamic term structure model for swap rates and infers the the swap rate distributions under the risk-neutral and physical measures. Section 5 investigates the economic determinants of the swap rate distributions. Section 6 considers a variety of robustness checks. Section 7 concludes, and two appendices contain additional information.

[8] See Gupta and Subrahmanyam (2005), Li and Zhao (2006), Jarrow, Li, and Zhao (2007), Deuskar, Gupta, and Subrahmanyam (2008), Trolle and Schwartz (2009), Li and Zhao (2009), and Deuskar, Gupta, and Subrahmanyam (2010).

[9] One drawback of the cap/floor market is that caplet prices are not quoted directly. Instead, one needs to strip caplet prices from quoted caps prices, which is a non-trivial exercise.

[10] See, among others, Ang and Piazzesi (2003), Gallmeyer, Hollifield, and Zin (2005), Ang, Piazzesi, and Wei (2006), Smith and Taylor (2009), Bekaert, Cho, and Moreno (2010), Bikbov and Chernov (2010), Chun (2010), and Joslin, Priebsch, and Singleton (2010).

[11] In contemporaneous work, Cieslak and Povala (2010) find that macro and liquidity factors are important determinants of Treasury market volatility. They study neither volatility risk premia or higher-order moments of interest rates.

2 The swaption cube data

A standard European swaption is an option to enter into a fixed versus floating forward starting interest rate swap at a predetermined rate on the fixed leg. A receiver swaption gives the right to enter a swap, receiving the fixed leg and paying the floating leg, while a payer swaption gives the right to enter a swap, paying the fixed leg and receiving the floating leg.[12] For instance, a two-year into ten-year, five percent payer swaption is the option to pay a fixed rate of five percent on a ten-year swap, starting two years from today.

The swaption cube is an object that shows how swaption prices vary along three dimensions: the maturities of the underlying swaps, the expiries of the options, and the option strikes. In the swaption cube data provided to us, prices are quoted for 5 different swap maturities (2, 5, 10, 20, and 30 years), 8 different option expiries (1, 3, 6, 9 months and 1, 2, 5, and 10 years), and up to 15 different strikes given by fixed distances to the forward swap rate ($\pm$ 400, $\pm$ 300, $\pm$ 200, $\pm$ 150, $\pm$ 100, $\pm$ 50, $\pm$ 25, and 0 basis points).[13] Hence, the swaption cube gives an extremely detailed view of the swaption market.

Swaptions trade over the counter (OTC) and typically via inter-dealer brokers. These act as intermediaries; they facilitate price discovery and transparency by communicating dealer interests and transactions, enhance liquidity, and allow financial institutions anonymity in terms of their trading activities. Our swaption cube data is from ICAP plc., which is the largest inter-dealer broker in the interest rate derivatives market, and as such provides the most accurate quotations.

We consider swaptions denominated in both USD and EUR, which are by far the most liquid markets.[14] Although the data is available daily, we use weekly (Wednesday) data to

[12] USD swaps exchange a fixed rate for a floating 3-month LIBOR rate, with fixed-leg payments made semi-annually, and floating-leg payments made quarterly. EUR swaps exchange a fixed rate for a floating 6-month EURIBOR rate, with fixed-leg payments made annually, and floating-leg payments made semi-annually. In both currencies, the daycount convention is 30/360 on the fixed leg, and Actual/360 on the floating leg.

[13] Prices are for out-of-the-money (OTM) swaptions, i.e. receiver swaptions, when the strike is less than the forward swap rate, and payer swaptions, when the strike is higher than the forward swap rate.

[14] To measure market concentration in each segment of the global OTC derivatives market, the Bank for International Settlements (BIS) computes a Herfindahl index defined as the sum of the squares of the market shares of each individual institution. The index ranges between 0 and 1, with its value increasing in market concentration. For OTC interest rate options (for which swaptions constitute the largest component) denominated in USD and EUR, the BIS computed a value of 0.0912 and 0.0638, respectively, as of December 2009; see, BIS (2010). As such, both markets are very competitive.

avoid potential weekday effects, and to ease the computational burden of the estimation. For the USD market, the data is from December 19, 2001 to January 27, 2010 (419 weeks), while for the EUR market, the data is from June 6, 2001 to January 27, 2010 (449 weeks). We apply various filters to the data; we eliminate obvious mistakes in the quotations and only consider options for which the price is larger than USD (EUR) 100 in case of a swap notional of USD (EUR) 1,000,000 (since according to market sources, quotes for extremely deep OTM swaptions are less reliable).[15] In total, we use 172,658 quotes in the USD market, and 172,500 quotes in the EUR market for our analyses.

While swaptions are quoted in terms prices, it is often more convenient to represent prices in terms of implied volatilities – either log-normal or normal.[16] Most market participants think in terms of normal implied volatilities, as these are more uniform across the swaption grid and more stable over time than log-normal implied volatilities. Therefore, in this paper, implied volatilities always refer to the normal type, unless otherwise stated.

3 A model independent analysis of the swaption cube

In this section, we analyze the swaption cube from a model independent perspective. For a given option expiry and swap maturity, conditional moments of the swap rate distribution (under the appropriate pricing measure) at a time horizon equal to the option expiry can be inferred from the implied volatility smile.[17] We analyze how conditional moments vary with option expiry, swap maturity, and across time.

[15]This implies that for a given underlying swap maturity, the range of strikes will increase with option expiry. For a given swap maturity and option expiry, the range of strikes will vary over time with the level of volatility and the level of the underlying forward swap rate (swaptions with negative strikes are obviously not quoted).

[16]The log-normal (or percentage) implied volatility is the volatility parameter that, plugged into the log-normal (or Black (1976)) pricing formula, matches a given price. The normal (or absolute or basis point) implied volatility is the volatility parameter that, plugged into the normal pricing formula, matches a given price.

[17]In principle, using the insight from Breeden and Litzenberger (1978), we could obtain the entire conditional density, rather than just conditional moments, of the swap rate from the implied volatility smile. Beber and Brandt (2006) and Li and Zhao (2009) study option-implied densities of Treasury futures prices and LIBOR rates, respectively. In practice, however, it is a non-trivial matter to obtain the conditional density from a finite number of option prices, and results may be quite sensitive to the choice of numerical scheme. In contrast, conditional moments can be recovered in a robust fashion.

3.1 Conditional moments of the swap rate distribution

Consider a fixed versus floating interest rate swap for the period T_m to T_n with a fixed rate of K. At every time T_j in a pre-specified set of dates $T_{m+1}, ..., T_n$, the fixed leg pays $\tau_{j-1}K$, where τ_{j-1} is the year-fraction between times T_{j-1} and T_j. The value of the swap at time $t < T_m$ (assuming a notional of one) is given by[18]

$$V_{m,n}(t) \quad = \quad P(t, T_m) - P(t, T_n) - KA_{m,n}(t), \tag{1}$$

where

$$A_{m,n}(t) = \sum_{j=m+1}^{n} \tau_{j-1} P(t, T_j), \tag{2}$$

and $P(t, T)$ denotes the time-t price of a zero-coupon bond maturing at time T. The time-t forward swap rate, $S_{m,n}(t)$, is the rate on the fixed leg that makes the present value of the swap equal to zero and is given by

$$S_{m,n}(t) = \frac{P(t, T_m) - P(t, T_n)}{A_{m,n}(t)}. \tag{3}$$

The forward swap rate becomes the spot swap rate at time T_m.

A payer swaption is an option to enter into an interest rate swap, paying the fixed leg at a predetermined rate and receiving the floating leg. Let $\mathcal{P}_{m,n}(t, K)$ denote the time-t value of a European payer swaption expiring at T_m with strike K on a swap for the period T_m to T_n.

[18] Here, and throughout the paper, we are implicitly assuming that LIBOR is the proper rate for discounting swap cash flows. In reality, inter-dealer swap and swaption contracts are virtually always collateralized, and cash flows should, in principle, be discounted using a risk-free rate. A number of recent papers have analyzed this issue (see, e.g., Fujii, Shimada, and Takahashi (2009), Mercurio (2009), Filipovic and Trolle (2010) and, for a more general treatment, Piterbarg (2010)), and it is generally agreed that one should use discount factors inferred from overnight index swaps (OIS), which are swaps that exchange a compounded overnight rate against a fixed rate. In principle, all formulas in the paper could be extended to take into account this extra complication. In practice, however, this extension is only possible for the very last part of the sample, where long-maturity OIS were actively traded. Consequently, and consistent with most papers on swaps and swaptions, we discount swap cash flows at LIBOR.

At expiration, the swaption has a payoff of[19]

$$\begin{aligned} V_{m,n}(T_m)^+ &= (1 - P(T_m, T_n) - K A_{m,n}(T_m))^+ \\ &= A_{m,n}(T_m) \left(S_{m,n}(T_m) - K\right)^+ . \end{aligned} \tag{4}$$

At time $t < T_m$, its price is given by

$$\begin{aligned} \mathcal{P}_{m,n}(t, K) &= E_t^{\mathbb{Q}} \left[e^{-\int_t^{T_m} r(s)ds} A_{m,n}(T_m) \left(S_{m,n}(T_m) - K\right)^+ \right] \\ &= A_{m,n}(t) E_t^{\mathbb{A}} \left[(S_{m,n}(T_m) - K)^+ \right], \end{aligned} \tag{5}$$

where $\mathbb{Q}$ denotes expectation under the risk-neutral measure, and $\mathbb{A}$ denotes expectation under the annuity measure associated with using $A_{m,n}(t)$ as numeraire.[20] The corresponding receiver swaption is denoted by $\mathcal{R}_{m,n}(t, K)$, and has a time-t price of

$$\mathcal{R}_{m,n}(t, K) = A_{m,n}(t) E_t^{\mathbb{A}} \left[(K - S_{m,n}(T_m))^+ \right]. \tag{6}$$

From (5) and (6) it is apparent that a receiver swaption can be viewed as a put option on a swap rate, whereas a payer swaption can be viewed as a call option on a swap rate.

Using the insights from Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003) it follows that for any fixed Z, we can write any twice continuously differentiable function of $S_{m,n}(T_m)$, $g(S_{m,n}(T_m))$, as

$$\begin{aligned} g(S_{m,n}(T_m)) &= g(Z) + g'(Z)(S_{m,n}(T_m) - Z) + \int_Z^{\infty} g''(K)(S_{m,n}(T_m) - K)^+ dK \\ &\quad + \int_0^Z g''(K)(K - S_{m,n}(T_m))^+ dK. \end{aligned} \tag{7}$$

Taking expectations under the annuity measure and setting $Z = S_{m,n}(t)$, we obtain an expres-

[19] EUR swaptions are typically cash-settled and have a payoff given by

$$(S_{m,n}(T_m) - K)^+ \sum_{j=m+1}^{n} \tau_{j-1} \frac{1}{(1 + S_{m,n}(T_m))^{\tau_{m,j}}},$$

where $\tau_{m,j}$ is the year-fraction between times T_m and T_j. The advantage of using this formula, rather than (4) is that counterparties only have to agree upon a single swap rate, rather than a complete set of discount factors, to compute the cash settlement value. In practice, the difference between the two payoff formulas is very small, and in the paper we use (4) also for EUR swaptions; see, e.g., Andersen and Piterbarg (2010) for further details.

[20] For a discussion of the annuity measure; see, e.g., Jamshidian (1997). Note that the annuity measure changes with m and n. To lighten notation, we have suppressed this dependence.

sion in terms of prices of out-of-the-money receiver and payer swaptions

$$E_t^{\mathbb{A}}\left[g(S_{m,n}(T_m))\right] = g(S_{m,n}(t)) + \frac{1}{A_{m,n}(t)}\Big(\int_{S_{m,n}(t)}^{\infty} g''(K)\mathcal{P}_{m,n}(t,K)dK + \int_0^{S_{m,n}(t)} g''(K)\mathcal{R}_{m,n}(t,K)dK\Big). \tag{8}$$

We can use this result to compute conditional moments of the swap rate distribution at a time horizon equal to the expiry of the option. First, by construction of the annuity measure, the conditional mean of the future swap rate distribution is simply the current forward swap rate:

$$\mu_t \equiv E_t^{\mathbb{A}}\left[S_{m,n}(T_m)\right] = S_{m,n}(t). \tag{9}$$

Then, using (8), we get the following expressions for conditional variance, skewness, and kurtosis of the future swap rate distribution:

$$\mathrm{Var}_t^{\mathbb{A}}\left(S_{m,n}(T_m)\right) = E_t^{\mathbb{A}}\left[\left(S_{m,n}(T_m)-\mu_t\right)^2\right] = \frac{2}{A_{m,n}(t)}\Big(\int_{S_{m,n}(t)}^{\infty}\mathcal{P}_{m,n}(t,K)dK + \int_0^{S_{m,n}(t)}\mathcal{R}_{m,n}(t,K)dK\Big) \tag{10}$$

$$\mathrm{Skew}_t^{\mathbb{A}}\left(S_{m,n}(T_m)\right) = \frac{E_t^{\mathbb{A}}\left[\left(S_{m,n}(T_m)-\mu_t\right)^3\right]}{\mathrm{Var}_t^{\mathbb{A}}\left(S_{m,n}(t,T_m)\right)^{3/2}} = \frac{\frac{6}{A_{m,n}(t)}\Big(\int_{S_{m,n}(t)}^{\infty}\left(K-S_{m,n}(t)\right)\mathcal{P}_{m,n}(t,K)dK + \int_0^{S_{m,n}(t)}\left(K-S_{m,n}(t)\right)\mathcal{R}_{m,n}(t,K)dK\Big)}{\mathrm{Var}_t^{\mathbb{A}}\left(S_{m,n}(t,T_m)\right)^{3/2}} \tag{11}$$

$$\mathrm{Kurt}_t^{\mathbb{A}}\left(S_{m,n}(T_m)\right) = \frac{E_t^{\mathbb{A}}\left[\left(S_{m,n}(T_m)-\mu_t\right)^4\right]}{\mathrm{Var}_t^{\mathbb{A}}\left(S_{m,n}(t,T_m)\right)^{2}} = \frac{\frac{12}{A_{m,n}(t)}\Big(\int_{S_{m,n}(t)}^{\infty}\left(K-S_{m,n}(t)\right)^2\mathcal{P}_{m,n}(t,K)dK + \int_0^{S_{m,n}(t)}\left(K-S_{m,n}(t)\right)^2\mathcal{R}_{m,n}(t,K)dK\Big)}{\mathrm{Var}_t^{\mathbb{A}}\left(S_{m,n}(t,T_m)\right)^{2}} \tag{12}$$

As discussed in the previous section, swaptions are only available for a finite set of strikes, while the formulas presume the existence of a continuum of strikes. Swaption prices corresponding to the required strikes in the scheme used for numerical integration are obtained by, first, linearly interpolating between the available normal implied volatilities, and then converting from implied volatilities to prices. For strikes below the lowest available strike, we use the implied volatility at the lowest strike. Similarly, for strikes above the highest available strike, we use the implied volatility at the highest strike. The approximation error caused by the extrapolation of implied volatilities is very small, since swaption prices are very low in the regions of strikes where extrapolation is necessary.[21]

3.2 Results

We now investigate how conditional moments vary with option expiry, swap maturity, and across time. Table 1 displays results for conditional volatility (the annualized standard deviation, measured in basis points) of the swap rate distribution for different swap maturities (tenors) and at different option expiries. It reports the sample means and, in parentheses, the sample standard deviations. At short option expiries, conditional volatility is, on average, a hump-shaped function of swap maturity, with the intermediate (5-year) segment of the swap term structure being the most volatile. For the shortest (2-year) swap maturity, conditional swap rate volatility is, on average, a hump-shaped function of option expiry, while for the longer swap maturities, it declines with option expiry. This pattern is also found in earlier studies that look at implied normal or log-normal ATM volatilities, without taking the implied volatility smile into account. It is consistent with a model in which innovations to the term structure of forward rates exhibits a hump-shape.

Conditional volatility exhibits significant variation over time. For instance, the conditional 1-year ahead distribution of the USD 10-year swap rate has a volatility that varies between 67 bp and 177 bp through the sample period (see solid line in Figure 1, Panel A). For a given swap maturity, the variation in volatility declines with option expiry, consistent with a model exhibiting mean-reverting stochastic volatility. Moreover, an unreported analysis shows that changes in volatility are largely unrelated to changes in the term structure. This is the "unspanned stochastic volatility" phenomenon, which is the subject of a number of recent papers.[22] A principal component (PC) analysis reveals large common variation in conditional volatility across the swaption matrix. For instance, in the USD market, the first PC explains 84 percent of the variation (while the second and third PCs explain 9 and 5 percent, respectively).

[21] We have experimented with different interpolation/extrapolation schemes and find that the results are very robust to the choice of scheme. The only exception is for swaptions with 10-year option expiries in times of high volatility, where swaptions outside of the available strike range do have some (small) values making the results slightly dependent on the extrapolation approach. The integrals are evaluated with the trapezoid scheme using 999 integration points for each integral. The first integral in each expression is truncated at $S_{m,n}(t) + 0.10$.

[22] This line of research was initiated by Collin-Dufresne and Goldstein (2002) and further evidence has been provided by Heidari and Wu (2003), Andersen and Benzoni (2010), Li and Zhao (2006, 2009), Trolle and Schwartz (2009), and Collin-Dufresne, Goldstein, and Jones (2009), among others.

Table 2 displays results for conditional skewness of the swap rate distribution for different swap maturities (tenors) and at different option expiries. Like the previous table, it reports the sample means and, in parentheses, the sample standard deviations. Conditional skewness tends to be positive on average, implying that for most points on the swaption matrix, OTM payer swaptions are more expensive than equivalently OTM receiver swaptions. At any given option expiry, conditional skewness, on average, declines with swap maturity to the point, where it is negative at the longest tenors – particularly in the EUR market. At the same time, for a given swap maturity, conditional skewness is an increasing and concave function of option expiry. If innovations to swap rates were independent and identically distributed, conditional skewness would decrease with option expiry by being proportional to the reciprocal of the square-root of the option expiry. Instead, the observed term structure of conditional skewness is consistent with volatility following a stochastic process with low to moderate degrees of mean reversion.[23]

Like conditional volatility, conditional skewness also exhibits significant variation over time. The variation is such that the sign of conditional skewness often changes. For example, skewness of the conditional 1-year ahead distribution of the USD 10-year swap rate varies between -0.38 and 0.87 through the sample period (see solid line in Figure 1, Panel B). A principal component analysis shows that conditional skewness also exhibits large common variation across the swaption matrix. In the case of the USD market, the first PC explains 85 percent of the variation (while the second and third PCs explain 7 and 2 percent, respectively).

Similar to volatility being only partially spanned by the term structure, an interesting question is the extent to which skewness is spanned by the term structure and/or volatility; that is, the extent to which "skewness risk" represents a separate source of risk. To investigate this issue, we initially extract the main PCs driving weekly changes in forward swap rates and the main PCs driving weekly changes in the conditional variance of the swap rate distributions. We use all the PCs explaining more than one percent of the variation ensuring that they summarize virtually all of the information in interest rates and volatility. Next, for each point on the swaption matrix, we regress changes in the conditional skewness of the swap

[23]The term structure of conditional skewness (and kurtosis) in stochastic volatility models is explored in Das and Sundaram (1999). In the Heston (1993) model, where variance follows a square-root process, the term structure of conditional skewness exhibits a hump shape. For parameter values often encountered in practice, the point of maximum conditional skewness occurs years into the future, implying that the term structure will often be increasing and concave over the set of option maturities actually observed.

rate distribution on the interest rate and volatility PCs (we also include the squared PCs in the regressions in an attempt to take non-linearities into account). The R^2s, which are reported in Table 3, are relatively small; in the USD market, ranging from 0.03 to 0.30 with an average of 0.14. Then, we factor analyze the covariance matrix of the 40 time series of regression residuals. The PCs of the residuals are, by construction, independent of those of interest rates and volatility. There is large common variation in the regression residuals, with the first PC explaining 78 percent of the variation in the USD market. Taken together, this strongly indicates that there is systematic variation in skewness which is largely independent of variation in interest rates and volatility.[24]

Due to space constraints, we only briefly summarize the results for conditional kurtosis of the swap rate distributions. The swap rate distributions are always leptokurtic. For a given swap maturity, the term structure of conditional kurtosis is hump-shaped with a peak between 5 and 10 years – again consistent with a stochastic volatility model with low to moderate degrees of mean reversion. While conditional kurtosis also exhibits some variation over time, it is less systematic across the swaption matrix. For instance, in the case of the USD market, the first PC explains only 42 percent of the variation. For this reason, in the rest of the paper, we will mainly focus on conditional volatility and skewness.

4 A dynamic term structure model for swap rates

In this section, we propose and estimate a dynamic term structure model, which is capable of matching the cross-sectional and time-series variation in the conditional moments of the swap rate distributions under the annuity measure. We then use the model to infer the conditional moments under the risk-neutral and physical measures.

[24]If the low R^2s in Table 3 were simply due to noisy data, we would not expect to find much common variation in the residuals. We have run several other regressions to check the robustness of the result. For instance, to take potential time-variation in the relationship between rates, volatility, and skewness into account, we also perform the analysis using a rolling window of 52 observations. That is, for each window we extract the first three PCs of rates and volatility, run the regressions, and factor analyze the residuals. The average R^2s are now somewhat larger, but we continue to find large common variation in the regression residuals.

4.1 The model

We first set up a general model under the risk-neutral measure and then find its dynamics under the physical and annuity measures. Subsequently, we discuss model features and the specifications that we estimate.

4.1.1 Dynamics under the risk-neutral measure

Let $P(t,T)$ denote the time-t price of a zero-coupon bond maturing at time T. We assume the following general specification for the dynamics of zero-coupon bond prices

$$\begin{aligned}
\frac{dP(t,T)}{P(t,T)} &= r(t)dt + \sum_{i=1}^{N} \sigma_{P,i}(t,T)\left(\sqrt{v_1(t)}dW_i^{\mathbb{Q}}(t) + \sqrt{v_2(t)}d\overline{W}_i^{\mathbb{Q}}(t)\right) && (13)\\
dv_1(t) &= (\eta_1 - \kappa_1 v_1(t) - \kappa_{12} v_2(t))dt + \sigma_{v1}\sqrt{v_1(t)}dZ^{\mathbb{Q}}(t) && (14)\\
dv_2(t) &= (\eta_2 - \kappa_{21} v_1(t) - \kappa_2 v_2(t))dt + \sigma_{v2}\sqrt{v_2(t)}d\overline{Z}^{\mathbb{Q}}(t), && (15)
\end{aligned}$$

where $W_i^{\mathbb{Q}}(t)$ and $\overline{W}_i^{\mathbb{Q}}(t)$, $i = 1, ..., N$, and $Z^{\mathbb{Q}}(t)$ and $\overline{Z}^{\mathbb{Q}}(t)$ denote Wiener processes under the risk-neutral measure. We allow for correlations between $Z^{\mathbb{Q}}(t)$ and $W_i^{\mathbb{Q}}(t)$, $i = 1, ..., N$, with correlations denoted by ρ_i. Similarly, we allow for correlations between $\overline{Z}^{\mathbb{Q}}(t)$ and $\overline{W}_i^{\mathbb{Q}}(t)$, $i = 1, ..., N$, and denote these correlations by $\overline{\rho}_i$. This is the most general correlation structure that preserves the tractability of the model.

Now, applying Ito's Lemma to (3) gives the dynamics of the forward swap rate under $\mathbb{Q}$

$$\begin{aligned}
dS_{m,n}(t) &= \left(-\sum_{i=1}^{N} \sigma_{S,i}(t,T_m,T_n)\sigma_{A,i}(t,T_m,T_n)(v_1(t)+v_2(t))\right)dt + \\
&\quad \sum_{i=1}^{N} \sigma_{S,i}(t,T_m,T_n)\left(\sqrt{v_1(t)}dW_i^{\mathbb{Q}}(t) + \sqrt{v_2(t)}d\overline{W}_i^{\mathbb{Q}}(t)\right), && (16)
\end{aligned}$$

where

$$\begin{aligned}
\sigma_{S,i}(t,T_m,T_n) &= \sum_{j=m}^{n} \zeta_j(t)\sigma_{P,i}(t,T_j) && (17)\\
\sigma_{A,i}(t,T_m,T_n) &= \sum_{j=m+1}^{n} \chi_j(t)\sigma_{P,i}(t,T_j), && (18)
\end{aligned}$$

and $\zeta_j(t)$ and $\chi_j(t)$ are (stochastic) weights that are given in Appendix A.

4.1.2 Dynamics under the physical measure

The dynamics under the physical probability measure $\mathbb{P}$ is obtained by specifying the market prices of risk that link the Wiener processes under $\mathbb{Q}$ and $\mathbb{P}$. We apply the following relatively standard specifications

$$dW_i^{\mathbb{P}}(t) = dW_i^{\mathbb{Q}}(t) - \lambda_i\sqrt{v_1(t)}dt, \;\; d\overline{W}_i^{\mathbb{P}}(t) = d\overline{W}_i^{\mathbb{Q}}(t) - \overline{\lambda}_i\sqrt{v_2(t)}dt, \;\; i = 1, ..., N \tag{19}$$

and

$$dZ^{\mathbb{P}}(t) = dZ^{\mathbb{Q}}(t) - \nu\sqrt{v_1(t)}dt, \;\; d\overline{Z}^{\mathbb{P}}(t) = d\overline{Z}^{\mathbb{Q}}(t) - \overline{\nu}\sqrt{v_2(t)}dt, \tag{20}$$

which implies the following dynamics of the forward swap rate under $\mathbb{P}$

$$\begin{aligned} dS_{m,n}(t) \;\; = \;\; & \left(-\sum_{i=1}^{N}\sigma_{S,i}(t,T_m,T_n)\left((\sigma_{A,i}(t,T_m,T_n)+\lambda_i)v_1(t) + (\sigma_{A,i}(t,T_m,T_n)+\overline{\lambda}_i)v_2(t)\right)\right)dt + \\ & \sum_{i=1}^{N}\sigma_{S,i}(t,T_m,T_n)\left(\sqrt{v_1(t)}dW_i^{\mathbb{P}}(t) + \sqrt{v_2(t)}d\overline{W}_i^{\mathbb{P}}(t)\right), \end{aligned} \tag{21}$$

where

$$dv_1(t) \;\; = \;\; \left(\eta_1 - \kappa_1^{\mathbb{P}}v_1(t) - \kappa_{12}v_2(t)\right)dt + \sigma_{v1}\sqrt{v_1(t)}dZ^{\mathbb{P}}(t) \tag{22}$$

$$dv_2(t) \;\; = \;\; \left(\eta_2 - \kappa_{21}v_1(t) - \kappa_2^{\mathbb{P}}v_2(t)\right)dt + \sigma_{v2}\sqrt{v_2(t)}d\overline{Z}^{\mathbb{P}}(t), \tag{23}$$

and $\kappa_1^{\mathbb{P}} = \kappa_1 - \sigma_{v1}\nu_v$ and $\kappa_2^{\mathbb{P}} = \kappa_2 - \sigma_{v2}\overline{\nu}_v$.

4.1.3 Dynamics under the annuity measure

As discussed in Section 3, for pricing swaptions it is convenient to work under the annuity measure, $\mathbb{A}$. Straightforward computations give the following dynamics of the forward swap rate under $\mathbb{A}$

$$dS_{m,n}(t) \;\; = \;\; \sum_{i=1}^{N}\sigma_{S,i}(t,T_m,T_n)\left(\sqrt{v_1(t)}dW_i^{\mathbb{A}}(t) + \sqrt{v_2(t)}d\overline{W}_i^{\mathbb{A}}(t)\right), \tag{24}$$

where

$$dv_1(t) \;\; = \;\; \left(\eta_1 - \kappa_1^{\mathbb{A}}v_1(t) - \kappa_{12}v_2(t)\right)dt + \sigma_{v1}\sqrt{v_1(t)}dZ^{\mathbb{A}}(t) \tag{25}$$

$$dv_2(t) \;\; = \;\; \left(\eta_2 - \kappa_{21}v_1(t) - \kappa_2^{\mathbb{A}}v_2(t)\right)dt + \sigma_{v2}\sqrt{v_2(t)}d\overline{Z}^{\mathbb{A}}(t), \tag{26}$$

and $\kappa_1^{\mathbb{A}} = \kappa_1 - \sigma_{v1}\sum_{i=1}^{N}\rho_i\sigma_{A,i}(t,T_m,T_n)$ and $\kappa_2^{\mathbb{A}} = \kappa_2 - \sigma_{v2}\sum_{i=1}^{N}\overline{\rho}_i\sigma_{A,i}(t,T_m,T_n)$. This leads to a fast and accurate Fourier-based pricing formula for swaptions derived in Appendix A.

4.1.4 Model features

The model has the potential to match the main stylized facts regarding conditional moments of the swap rate distributions reported in Section 3. A specification with only one volatility process could capture the cross-sectional variation in average conditional swap rate volatility and skewness. The former requires specifying the zero-coupon bond volatility functions, $\sigma_{P,i}(t,T)$, such that the intermediate part of the term structure is the most volatile. The latter is a consequence of a mean-reverting volatility processes combined with the possibility of correlation between innovations to the term structure and volatility. Clearly, a specification with only one volatility process would also be consistent with variation in conditional swap rate volatility over time, including the unspanned stochastic volatility phenomenon. However, it would not be consistent with two important properties of conditional swap rate skewness; first, its significant variation over time – in particular the switch in the sign of skewness – and, second, the fact that skewness is largely unspanned by rates and volatility. In order to match these two properties, we need at least two volatility processes along with certain parameter restrictions.

To illustrate how a specification with two volatility processes could be consistent with the first property, consider the case of $N = 1$. In this case, the instantaneous correlation between innovations to the forward swap rate, $S_{m,n}(t)$, and its variance, $\sigma_{S,1}(t, T_m, T_n)(v_1(t) + v_2(t))$, is given by

$$\frac{\sigma_{v1}\rho_1 v_1(t) + \sigma_{v2}\overline{\rho}_1 v_2(t)}{\sqrt{v_1(t) + v_2(t)}\sqrt{\sigma_{v1}^2 v_1(t) + \sigma_{v2}^2 v_2(t)}}, \tag{27}$$

which depends on the relative magnitudes of $v_1(t)$ and $v_2(t)$ and may switch sign if ρ_1 and $\overline{\rho}_1$ have opposite signs. Conditional swap rate skewness, which depends on this correlation, may therefore also switch sign.[25] Obviously, with $N > 1$, one can generate richer dynamics in conditional skewness.

For a specification with two volatility processes to also be consistent with the unspanned stochastic skewness property, we impose that the drift and diffusion parameters of the volatility processes are identical.[26] Then, shocks to $v_1(t)$ and $v_2(t)$ have the same effect on conditional

[25] Carr and Wu (2007) and Christoffersen, Heston, and Jakobs (2009) use a similar technique to generate stochastic skewness in currency and stock return distributions.

[26] In this case (27) reduces to a simple weighted average of ρ_1 and $\overline{\rho}_1$

$$\rho_1 w(t) + \overline{\rho}_1(1 - w(t)), \quad w(t) = \frac{v_1(t)}{v_1(t) + v_2(t)}. \tag{28}$$

swap rate volatility and conditional swap rate skewness may vary independently of conditional swap rate volatility.

4.1.5 Model specifications

It is well known that the term structure of interest rates is driven by three factors, and, accordingly, we set $N = 3$ in all specifications. For the bond price volatility functions in (13), we note that we can equally well specify the volatility functions for instantaneous forward rates, since the two are related by $\sigma_{P,i}(t,T) = -\int_t^T \sigma_{f,i}(t,u)du$. As it is generally easier to relate to interest rate volatility than bond price volatility, we show both. The specifications we use are

$$
\begin{aligned}
\sigma_{f,1}(t,T) &= \alpha_1 e^{-\xi(T-t)} & \sigma_{P,1}(t,T) &= \tfrac{\alpha_1}{\xi}\left(e^{-\xi(T-t)} - 1\right) \\
\sigma_{f,2}(t,T) &= \alpha_2 e^{-\gamma(T-t)} & \sigma_{P,2}(t,T) &= \tfrac{\alpha_2}{\gamma}\left(e^{-\gamma(T-t)} - 1\right) \\
\sigma_{f,3}(t,T) &= \alpha_3 (T-t) e^{-\gamma(T-t)} & \sigma_{P,3}(t,T) &= \tfrac{\alpha_3}{\gamma^2}\left(e^{-\gamma(T-t)} - 1\right) + \tfrac{\alpha_3}{\gamma}(T-t)e^{-\gamma(T-t)}.
\end{aligned}
$$

The second and third are the "slope" and "curvature" factor loadings proposed by Nelson and Siegel (1987), while the first becomes their "level" factor loading in the limit $\xi \to 0$. These factor loadings are popular in the term structure literature as they parsimoniously capture the predominant shocks to the term structure. The reason behind modifying the first factor loading relative to Nelson and Siegel (1987) is that this allows us to express the dynamics of the term structure in terms of a finite dimensional affine state vector, making it possible to estimate the model with well-established techniques from the vast affine term structure literature (in reality, ξ is estimated close to zero, which implies that the first factor still acts like a level factor). The affine representation of the model can be derived along the lines of Trolle and Schwartz (2009) and, due to space constraints, is given in a separate appendix available upon request.[27]

For the volatility dynamics, we consider two specifications. In what we denote the SV1 specification, we assume that there is only one volatility factor, $v_1(t)$. In what we call the SV2 specification, we assume that there are two volatility factors, $v_1(t)$ and $v_2(t)$, but impose that $\eta_1 = \eta_2$, $\kappa_1 = \kappa_2$, $\kappa_{12} = \kappa_{21} = 0$, and $\sigma_{v1} = \sigma_{v2}$, in which case the volatility factors only differ

[27] In principle, the model is time-inhomogeneous and fits the initial term structure by construction. For the purpose of estimation it is more convenient to work with the model's time-homogeneous counterpart. We therefore assume that the initial forward rate curve is flat and equal to a constant, φ, which is an additional parameter of the model. One can show that φ equals the infinite-maturity forward rate.

in terms of the their correlation with the term structure factors.[28] In Section 6.1, we consider more general specifications.

Coupled with the market price of risk specification discussed above, SV1 (SV2) has 11 (14) risk-neutral parameters and 6 (8) market price of risk parameters, which is well within what is often encountered in the empirical term structure literature. Given the vast amount data at our disposal, the model is quite tightly parameterized.

4.1.6 Maximum-likelihood estimation

We estimate the two specifications on all available swap rates and swaptions using maximum-likelihood in conjunction with Kalman filtering. Critical to estimating the model on all swaptions across time, and across all option expiries, swap maturities, and strikes, is the existence of an efficient pricing formula. Due to the non-linearities in the relationship between observations and state variables, we apply the non-linear unscented Kalman filter.[29] Details are provided in Appendix B.

4.2 Results

Table 4 displays parameter estimates of the SV1 and SV2 specifications in the two markets.[30] [31] One thing to note about the estimates of the SV2 specification is the opposite sign on ρ_i and $\overline{\rho}_i$, for most i, which is consistent with stochastic skewness that may switch sign. When $v_1(t)$ is high relative to $v_2(t)$, the swap rate distributions will be skewed towards lower interest rates, whereas when $v_1(t)$ is low relative to $v_2(t)$, the swap rate distributions will be skewed towards

[28]For the specifications to be identified, we set $\sigma_{v1} = 1$ in SV1, and $\sigma_{v1} = \sigma_{v2} = 1$ in SV2.

[29]The unscented Kalman filter has gained popularity in recent years as an alternative to the more standard extended Kalman filter. Christoffersen et al. (2009) perform an extensive Monte Carlo experiment, which shows that the unscented Kalman filter significantly outperforms the extended Kalman filter in the context of estimating dynamic term structure models with swap rates. Their results most likely carry over to our context, where swaptions are also used in the estimation.

[30]Since we are mainly interested in the higher-order moments of the swap rate distributions, and to preserve space, we have left out the estimates of premia associated with interest rate risk (λ_1, λ_2, λ_3, $\overline{\lambda}_1$, $\overline{\lambda}_2$, and $\overline{\lambda}_3$) from the table. These estimates are in a separate table available upon request.

[31]The asymptotic covariance matrix of the estimated parameters is computed from the outer-product of the first derivatives of the likelihood function. Theoretically, it would be more appropriate to compute the asymptotic covariance matrix from both the first and second derivatives of the likelihood function. In reality, however, the second derivatives of the likelihood function are somewhat numerically unstable.

higher interest rates. Another thing to note is that the premia associated with volatility risk, ν and $\overline{\nu}$, are estimated to be significant and negative. This issue is explored further below.

From the filtered state variables, we compute fitted values of interest rates and swaptions, as well as pricing errors. For swaptions, the pricing errors are the differences between fitted and actual normal implied volatilities. On each day in the sample, we then compute the root mean squared pricing errors (RMSEs) of the interest rates and swaptions available on that day. This way we construct time series of RMSEs for each model specification. The fit to interest rates is very good and very similar for the two model specifications. In the USD market, for instance, the mean RMSE is about 5 bp. This finding is not surprising, since both specifications have three term structure factors and similar estimates for the factor loadings. More interesting is the fit to swaptions. The first row of Table 5 displays the mean of the RMSEs. It also displays the mean difference in RMSEs between the two model specifications, and the associated t-statistics. The difference in overall pricing errors is strongly significant. For instance, in the USD market, the mean RMSE drops from 5.44 bp to 4.58 bp.

As an example of the improved fit, Figure 2 shows actual and fitted time-series of the USD normal implied volatility smile of the 1-year option on 10-year swap rate. Clearly, the SV2 specification matches the variation in the implied volatility smile much better than the SV1 specification.

To understand the improvement in pricing, we compute volatility (annualized), skewness, and kurtosis of the future swap rate distributions (under $\mathbb{A}$) implied by the two model specifications. This is done using the fitted swaption prices and the same interpolation/extrapolation scheme as in Section 3. On each day in the sample, we then compute RMSEs across all tenor – option expiry categories, where the errors are now the differences between actual and fitted moments, rather than implied volatilities. This way we construct time series of RMSEs of volatility, skewness, and kurtosis for each model specification. The second to fourth row of Table 5 display the means of these RMSEs as well as the mean differences in RMSEs between the two model specifications and the associated t-statistics. The mean volatility RMSEs are very similar for the two model specifications, and the difference is not very significant. This is expected, given that we impose that the two volatility state variables in the SV2 specification only differ in terms of their correlation parameters. In contrast, the mean skewness RMSE is significantly lower for the SV2 specification. For instance, in the USD market the mean skewness RMSE is 0.21 and 0.05, for the SV1 and SV2 specification, respectively.[32]

To visualize the improvement in the fit to the moments, Figure 1 displays time-series of conditional volatility and skewness of the 1-year ahead distribution of the USD 10-year swap rate (again under $\mathbb{A}$). While both model specifications have very similar fit to conditional volatility, they differ markedly in their fit to conditional skewness. The SV1 specification has too little variation in conditional skewness, and model-implied skewness is in fact negatively correlated with actual skewness. In contrast, conditional skewness of the SV2 specification tracks actual skewness closely – in particular, it captures the switches between positive and negative skewness.

Table 6 elaborates on the relative fit to skewness. It displays the mean differences in absolute skewness errors for the two specifications within each tenor – option expiry category, and the associated t-statistics. It shows that the SV2 specification entails a significant improvement in the fit to skewness across virtually the entire swaption matrix.

For each specification, we also run the unspanned stochastic skewness regressions from Section 3; i.e. regressing changes in the conditional skewness on the main PCs driving weekly changes in forward swap rates and conditional volatilities. For the SV1 specification, the R^2s are large, ranging from 0.48 to 0.91 with an average of 0.72, implying that skewness risk is largely spanned. In contrast, the SV2 specification generates a degree of unspanned stochastic skewness which is close to that observed in the data with R^2s ranging from 0.06 to 0.21 with an average of 0.13 (compared with 0.14 in the data).

Having established that the SV2 specification provides a good fit to the cross-sectional and time-series variation in the conditional moments (particularly volatility and skewness) of the swap rate distributions under $\mathbb{A}$, we use the model to infer the conditional moments under the risk-neutral measure $\mathbb{Q}$ and the physical measure $\mathbb{P}$. This is done by simulating the distributions of future swap rates, using the formulas in Sections 4.1.1 and 4.1.2, and from the simulated distributions computing the moments.[33] We then define volatility risk premia as the differences between conditional swap rate volatilities under $\mathbb{P}$ and $\mathbb{Q}$. Similarly, we define skewness risk premia as the differences between conditional swap rate skewness under $\mathbb{P}$ and $\mathbb{Q}$.

Table 7 shows, for each point on the swaption matrix, the average volatility risk premium and, in parentheses, the standard deviation of the volatility risk premium. Average volatility

[32]Note that the mean kurtosis RMSE is also significantly lower for the SV2 specification.

[33]We use 50.000 simulations and anti-thetic variates.

risk premia are negative across the matrix for both currencies, implying that conditional volatility is typically higher under $\mathbb{Q}$ than $\mathbb{P}$, which is consistent with what Duarte, Longstaff, and Yu (2007) find for the USD cap/floor market.[34] Volatility risk premia are somewhat more negative in the USD market than in the EUR market.

Table 8 shows results for skewness risk premia. Average skewness risk premia are negative across the matrix for both currencies, implying that the conditional $\mathbb{Q}$-distribution is typically skewed more towards higher interest rates than the conditional $\mathbb{P}$-distribution. Skewness risk premia are of the same magnitude in the two markets.

5 Fundamental drivers of the swap rate distributions

While we have used a reduced-form dynamic term structure model to infer the conditional moments of the swap rate distributions under $\mathbb{P}$ and $\mathbb{Q}$, ultimately we are interested in understanding the fundamental drivers of these conditional moments. For this purpose, we regress volatility and skewness of the physical swap rate distributions as well as volatility and skewness risk premia on a number of variables motivated by economic theory and prior results in the literature. We are primarily interested in the effects of macro-economic uncertainty, which we proxy by dispersion and skewness of agents' perceived probability distributions for future real GDP growth and inflation. But we also control for other factors that may have an effect on swap rate distributions, including moments of the equity index return distribution, a measure of market-wide illiquidity, and a measure of refinancing activity. These variables are described in more detail below.

5.1 Explanatory variables

5.1.1 Moments of agents' belief distributions for future real GDP growth and inflation

A number of equilibrium pricing models, primarily related to equity derivatives, imply that volatility and volatility risk premia are increasing in uncertainty and/or disagreement among

[34] Our model likely understates the magnitude of volatility and skewness risk premia for very short horizons. It is a well-known deficiency of stochastic volatility models that, for plausible parameter values, they cannot generate a sufficiently large wedge between the $\mathbb{P}$ and $\mathbb{Q}$ distributions over very short horizons. To do so, we would need to add a jump process to our model, which is possible, but beyond the scope of the present paper.

agents about fundamentals. For instance, in long run risk models, where agents have preferences for early resolution of uncertainty, and macro-economic uncertainty is stochastic, economic uncertainty is a priced source of risk.[35] That is also the case in models where agents have incomplete information and face fundamentals subject to regime switches.[36] Similarly, in models where agents have incomplete information and heterogeneous beliefs, disagreement among agents is a priced source of risk.[37]

Motivated by these papers, we investigate the extent to which agents' perceptions about macro-economic risks affect swap rate distributions. We focus on the perceived risks to future real GDP growth and inflation, which are among the most important fundamental determinants of interest rates. For this purpose, we use the quarterly survey of professional forecasters (SPF) conducted in the US by the Federal Reserve Bank of Philadelphia and in the Eurozone by the ECB. The SPF is unique, because participants are asked to assign a probability distribution to their forecasts for real GDP growth and inflation. We aggregate the probability distributions of the individual respondents, and compute dispersion (i.e., standard deviation) and skewness of the aggregate distributions of future real GDP growth and inflation.[38] Note that these aggregate belief distributions take both individual uncertainty and disagreement among agents into account.[39]

5.1.2 Moments of the equity index return distribution

Numerous papers have documented that equity and fixed-income markets are interconnected. We therefore investigate the extent to which the characteristics of the equity index return distribution have an impact on the swap rate distributions. Specifically, we consider the S&P

[35]See, e.g., Eraker and Shaliastovich (2008), Bollerslev, Tauchen, and Zhou (2009), Bollerslev, Sizova, and Tauchen (2009), Drechsler and Yaron (2009), and Shaliastovich (2009).

[36]See, e.g., David and Veronesi (2002, 2009).

[37]See, e.g., Buraschi and Jiltsov (2006) and Buraschi, Trojani, and Vedolin (2009).

[38]Participants are asked to provide probability distributions for the current and following calendar year. We follow Bekaert and Engstrom (2009) in weighting the probability distributions so as to maintain a 1-year-ahead forecast horizon. Another issue is that in forming their probability distributions, respondents are asked to attach probabilities to the outcome being in specific ranges. When computing moments of the aggregate distributions, we assume that the probability for a given range relates to the mid-point of that range.

[39]For instance, one can show that the variance of the aggregate distribution is equal to the average variance of the individual distributions (i.e. individuals' uncertainty) plus the variance of the point estimates (i.e. disagreement), see Giordani and Soderlind (2003).

500 index in the USD market and the Eurostoxx 50 in the EUR market and compute volatility and skewness of the risk-neutral return distributions in a model independent way, using the formulas in Bakshi, Kapadia, and Madan (2003). As in Section 3, this involves integrating over options with different strikes. We obtain risk-neutral moments for return horizons corresponding to the option expiries that are traded, and we use the first principal component of volatility and skewness, respectively, in the regressions.[40] Our measure of volatility in the USD market has a very high correlation (above 0.98) with the VIX index, which has been used in numerous studies as a proxy for overall financial market volatility, or as a sentiment indicator.

5.1.3 Market-wide illiquidity

A number of papers show that liquidity affects derivatives prices.[41] Unfortunately, our data set does not include bid-ask spreads or other measures, such as market depth, that can be used to construct liquidity measures at the level of individual contracts. Instead, we investigate the effect of liquidity at the market-wide level. As a proxy for market-wide illiquidity, we use the spread between the 3-month overnight index swap (OIS) rate and the 3-month Treasury bill yield (for the EUR market, we use the German counterpart to the 3-month Treasury bill). Since an OIS is a measure the expected average overnight rate during the life of the swap and is virtually free of credit and counterparty risk, the spread is a fairly clean proxy for illiquidity, as also observed by Krishnamurthy (2010).[42]

[40] For the S&P 500 index, the first principal component explains 98 (92) percent of the variation in volatility (skewness) across the option expiries. For the Eurostoxx 50, the numbers are similar.

[41] For instance, in the related market for caps and floors, Deuskar, Gupta, and Subrahmanyam (2010) find that liquidity, as proxied by bid-ask spreads, impacts prices. Other studies include Cetin et al. (2006) and Bongaerts, de Jong, and Driessen (2010), who provide theory and evidence in support for liquidity having an impact on the pricing of stock options and credit default swaps.

[42] In the USD market, an OIS is referenced to the overnight federal funds rate, while in the EUR market, it is referenced to the euro overnight index average (EONIA) rate. The swap contract itself is fully collateralized, making credit and counterparty risk negligible. As an alternative illiquidity proxy, we have used the spread between yields on off-the-run and on-the-run government bonds of comparable maturities but found similar results.

5.1.4 Refinancing activity

Several papers find that derivatives prices are affected by supply and demand.[43] In the swaption market, dealers absorb or redistribute supply and demand for volatility. From a dealer perspective, much of the supply of volatility emanates from issuance of callable debt by financial institutions and large corporations. A significant part of this debt is swapped into floating rate payments with the embedded optionality often passed on to dealers. In the USD market, an important demand for volatility comes from investors in MBSs, who actively hedge the negative convexity risk stemming from the prepayment options embedded in fixed rate mortgages. Active hedgers include mortgage giants Federal National Mortgage Association ("Fannie Mae") and the Federal Home Loan Mortgage Corporation ("Freddie Mac")[44] as well as mortgage hedge funds.

Since there is no quantitative information on the flows in the swaption market, it is difficult to estimate the extent to which demand and supply affects pricing. Like previous papers, we focus on the effect of refinancing activity. Duarte (2008) uses a measure of the average refinancing incentive in the mortgage universe as a proxy for the swaption demand by active hedgers in the MBS market and finds that hedging pressures have a significant impact on realized and implied volatility levels. We take a simpler route and use the Mortgage Bankers Association (MBA) Refinancing Index, which is a weekly measure of refinancing activity.[45]

5.2 Results

We face two issues regarding the regressions. First, in principle we could run regressions for volatility, skewness, and associated risk premia in each tenor – option expiry category. However, as these quantities are highly correlated across the swaption matrix, we instead run regressions using cross-sectional averages of volatility, skewness, and associated risk premia.[46]

[43]For instance, Bollen and Whaley (2004) and Gârleanu, Pedersen, and Poteshman (2009) find demand effects in the pricing of equity options.

[44]Fannie Mae and Freddie Mac were placed into conservatorship in September 2008. However, they were allowed to increase (and actively hedge) their total portfolio of retained mortgages to USD 1.7tn by 2009. Therefore, they continue to play a role in the interest rate derivatives market.

[45]The same proxy is used by Li and Zhao (2009) to study the effect of refinancing activity on the cap/floor market.

[46]Very similar results are obtained if, instead of using cross-sectional averages, we use the first principal components of volatility, skewness, and associated risk premia.

In other words, our focus is on understanding the overall time-series variation, rather than the cross-sectional variation. Second, our proxies for macro-economic uncertainty are only available at a quarterly frequency, while the remaining variables are available at a weekly frequency. To make use of all the information in the data, we run MIDAS-type regressions[47] of the following form

$$\begin{aligned} y_t &= \beta_0 + \beta_1 f(\theta,\tau) GDPvol_{t_q} + \beta_2 f(\theta,\tau) GDPskew_{t_q} + \beta_3 f(\theta,\tau) INFvol_{t_q} + \beta_4 f(\theta,\tau) INFskew_{t_q} + \\ & \quad \beta_5 EQvol_t + \beta_6 EQskew_t + \beta_7 ILLIQ_t + \beta_8 REFI_t + \epsilon_t, \end{aligned} \tag{29}$$

where $\tau = t - t_q$ is the time between the weekly observation at t and the most recent quarterly observation at t_q, and y_t is the cross-sectional average of either physical volatility, volatility risk premia, physical skewness, or skewness risk premia. The function $f(\theta,\tau)$ weighs the quarterly observations according to their distance from t. We assume the following simple functional form $f(\theta,\tau) = \exp(-\theta\tau)$; i.e., the weights are exponentially declining in τ. We also assume that the same weighing function applies to all four quarterly series. The MIDAS regression model is estimated by non-linear least squares, and Tables 9 and 10 display the results for the USD and EUR markets, respectively.

Consider first the relationships between the swap rate distributions and the characteristics of agents' belief distributions for the macro-economy. In the USD market, physical swap rate volatility depends significantly and positively on the dispersion of agents' GDP belief distribution, while volatility risk premia have a significantly negative dependance on the dispersion of agents' GDP beliefs. That is, an increase in the perceived uncertainty about future real GDP growth increases risk-neutral swap rate volatility more than physical volatility. This is consistent with the various types of equilibrium models, mentioned above, in which an increase in uncertainty and/or disagreement among agents about fundamentals increases both risk-neutral and physical volatility as well as the wedge between the two, since uncertainty and/or disagreement directly enters the stochastic discount factor. Also, consistent with intuition, physical swap rate skewness depends positively and significantly on the skewness (and, to a lesser extent, the dispersion) of agents' GDP beliefs; i.e., when the perceived risk to future real GDP growth is more skewed to the upside, the physical swap rate distributions tend to be skewed towards higher interest rates.

[47]MIxed DAta Sampling regressions have become popular in the econometrics literature following the initial publications by Ghysels, Santa Clara, and Valkanov (2005, 2006).

In the EUR market, physical swap rate volatility depends significantly and positively on the dispersion of agents' inflation belief distribution (and, more weakly, on the skewness of GDP beliefs), while volatility risk premia have a significantly negative dependance on the dispersion of inflation (and, to a lesser extent, GDP) beliefs. That is, perceived uncertainty about future inflation appear to be a relatively more important state variable for volatility and volatility risk premia than perceived uncertainty about future real GDP growth. Physical swap rate skewness depends positively and significantly on the skewness of agents' inflation beliefs (and, more weakly, on the dispersion of inflation beliefs), while skewness risk premia have a significantly negative dependence on the skewness of inflation beliefs (there is also a weak dependence on the dispersion of GDP and inflation beliefs). The latter result possibly reflects agents' dislike for high inflation states; when the physical likelihood of high interest rates due to high inflation increases, the risk-neutral likelihood of these states increases even more, increasing risk-neutral skewness relative to physical skewness.

It is striking that the characteristics of agents' inflation beliefs are the main determinant of EUR swap rate distributions, while the USD swap rate distributions are mostly related to the characteristics of agents' GDP beliefs. One likely explanation is differences in monetary policy objectives in the two economies. The primary policy goal of the European Central Bank is to maintain price stability, whereas the Federal Reserve has a dual mandate of maximum employment and price stability, leading it to place relatively more emphasis on expectations for real GDP growth when setting interest rates.[48]

Next, consider the relationships between the swap rate distributions and the equity index return distribution, market-wide illiquidity, and refinancing activity. In both markets, equity return volatility has a significantly positive effect on physical swap rate volatility and a significantly negative effect on volatility risk premia, while equity return skewness has a significantly positive effect on physical swap rate skewness. These results underscore the integration between equity and fixed income markets.

Market-wide illiquidity has a significantly positive effect on physical swap rate volatility and, in the USD market, a significantly negative effect on volatility risk premia. That is, a deterioration in liquidity increases risk-neutral swap rate volatility more than physical volatil-

[48] The different policy objectives of the two central banks were clearly illustrated on July 2, 2008, when the ECB, despite signs of financial stress and weakening growth, raised its benchmark rate in a bid to attack inflation. At that point, the Federal Reserve had already reduced its benchmark rate significantly to support economic growth in the face of the financial crisis.

ity.[49]

Finally, in the USD market, refinancing activity has a significantly positive effect on physical swap rate volatility and a (marginally) significantly negative impact on volatility risk premia. Compared with the results in Duarte (2008) (and Li and Zhao (2009) for the cap/floor market), the effect of refinancing activity is relatively modest. This may be due to differences in data, sample period, and methodology, but may also, in part, reflect the Federal Reserve's USD 1.25 trillion program to purchase MBSs, initiated in late 2008. Since the Federal Reserve does not engage in convexity hedging, its massive involvement in the MBS market reduces the effect of refinancing activity on the swaption market.[50] Refinancing activity, which is a U.S. phenomenon, has virtually no effect on the swap rate distributions in the EUR market.

6 Robustness checks

Our inferences in Sections 4 and 5 are clearly model dependent. In this final section, we consider the robustness of our results to alternative specifications for the term structure model, the risk premia, and the regressions.

6.1 Alternative term structure model specifications

First, we relax the constraints in the SV2 specification that the drift and diffusion parameters of the volatility processes are identical. We denote this specification SV2gen. Compared to the SV2 specification, the speed of mean-reversion decreases for $v_1(t)$ and increases for $v_2(t)$, while the correlation parameters ρ_i and $\overline{\rho}_i$ become more similar for all i. As a consequence, the fit to conditional swap rate volatilities improves, while the fit to conditional swap rate skewness deteriorates.[51] Also, skewness risk is largely spanned in the SV2gen specification, with the unspanned stochastic skewness regressions producing an average R^2 of 0.76. Hence,

[49] Note that there is a relatively high correlation between our illiquidity proxy and equity market volatility. Leaving out equity market volatility from the regressions increases the importance of market-wide illiquidity for physical swap rate volatility and volatility risk premia.

[50] This appear to be a consensus view among market participants. For instance, in a recent research report, Barclays Capital (2010) concludes that "...the Fed portfolio serves as a dampener on the option bid from MBS portfolios".

[51] Compared to the SV2 specification, in the USD market, the mean RMSE of swaption implied volatilities decreases from 4.58 to 4.22. The mean RMSE of conditional swap rate volatilities decreases from 4.83 to 3.97, while the mean RMSE of conditional swap rate skewness increases from 0.05 to 0.16.

the SV2gen specification trades a worse fit to conditional skewness for a better fit to conditional volatility. As we are interested in capturing the dynamics of both moments, we therefore do not consider this specification in more detail.

As an alternative, we instead extend the SV2 specification with an additional process, which captures low-frequency variation in $v_1(t)$ and $v_2(t)$. Specifically, we set $\eta_1 = \eta_2 = \eta(t)$, where $\eta(t)$ follows the stochastic process

$$d\eta(t) \quad = \quad (\overline{\eta} - \kappa_\eta \eta(t))\, dt + \sigma_\eta \sqrt{\eta(t)} d\widetilde{Z}^{\mathbb{Q}}(t), \tag{30}$$

where $\widetilde{Z}^{\mathbb{Q}}(t)$ is another Wiener process uncorrelated with $Z^{\mathbb{Q}}(t)$ and $\overline{Z}^{\mathbb{Q}}(t)$ (as well as with $W_i^{\mathbb{Q}}(t)$ and $\overline{W}_i^{\mathbb{Q}}(t)$, $i = 1, ..., N$).[52] We denote this specification SV3. The estimated parameters of the $v_1(t)$ and $v_2(t)$ processes are similar to those in the SV2 specification, except that the speed of mean-reversion is somewhat faster, while the process for $\eta(t)$ is estimated to have fairly slow mean-reversion. The SV3 specification succeeds in improving the fit to conditional volatility, while preserving the fit to conditional skewness.[53] It also preserves the unspanned nature of skewness risk with the unspanned stochastic skewness regressions producing R^2s only slightly larger than those of the SV2 specification. However, despite its better fit, the time-series for the cross-sectional averages of volatility and skewness of the physical swap rate distributions as well as volatility and skewness risk premia correspond rather closely to those of the SV2 specification. For this reason, and because we value parsimony, we base our inferences on that specification.

6.2 Alternative risk premia specifications

We have experimented with the following, more flexible, market price of risk specification, first suggested by Cheredito, Filipovic, and Kimmel (2007) and Collin-Dufresne, Goldstein, and Jones (2009):

$$dZ^{\mathbb{P}}(t) = dZ^{\mathbb{Q}}(t) - \frac{\nu_0 + \nu v_1(t)}{\sqrt{v_1(t)}} dt, \;\; d\overline{Z}^{\mathbb{P}}(t) = d\overline{Z}^{\mathbb{Q}}(t) - \frac{\overline{\nu}_0 + \overline{\nu} v_2(t)}{\sqrt{v_2(t)}} dt. \tag{31}$$

[52]Via a straightforward extension of the results in Appendix A, swaptions can also be priced quasi-analytically in this specification. To get the dynamics for $\eta(t)$ under the physical probability measure $\mathbb{P}$, we apply the following market price of risk specification: $d\widetilde{Z}^{\mathbb{P}}(t) = d\widetilde{Z}^{\mathbb{Q}}(t) - \widetilde{\nu}\sqrt{\eta(t)}dt$.

[53]Indeed, compared to the SV2 specification, in the USD market, the mean RMSE of swaption implied volatilities decreases from 4.58 to 3.29, with the mean RMSE of conditional swap rate volatilities decreasing from 4.83 to 3.49, and the mean RMSE of conditional swap rate skewness essentially unchanged.

For $\nu_0 = \overline{\nu}_0 = 0$, this reduces to the specification in (20).[54] In practice, the additional flexibility in (31) has little impact on the results, since, in all our estimation trials, ν_0 and $\overline{\nu}_0$ are not significantly different from zero, and the estimates of ν and $\overline{\nu}$ are similar to those reported in Table 4.[55] Also, conceptually we prefer (20) to (31), since in the latter specification, market prices of risk may become arbitrarily large as the volatility processes approach their zero boundaries, which is not particularly intuitive.

6.3 Alternative regression specifications

The MIDAS literature has proposed more flexible weighing functions than the simple one in Section 5.2. We have experimented with several of these, but found the results quite robust to the choice of (sensible) weighing scheme.

More importantly, we also run the regressions in quarterly differences, i.e.

$$\begin{aligned} \Delta y_{t_q} &= \beta_0 + \beta_1 \Delta GDPvol_{t_q} + \beta_2 \Delta GDPskew_{t_q} + \beta_3 \Delta INFvol_{t_q} + \beta_4 \Delta INFskew_{t_q} + \\ &\quad \beta_5 \Delta EQvol_{t_q} + \beta_6 \Delta EQskew_{t_q} + \beta_7 \Delta ILLIQ_{t_q} + \beta_8 \Delta REFI_{t_q} + \epsilon_{t_q}, \end{aligned} \tag{32}$$

where Δy_{t_q} is the quarterly change in the cross-sectional average of either physical volatility, volatility risk premia, physical skewness, or skewness risk premia. While this entails discarding information, it may be more robust than the MIDAS specification. Tables 11 and 12 display the results for the USD and EUR markets, respectively. The results are generally consistent with those obtained from the MIDAS regressions. In particular, we continue to find that the swap rate distributions are related to the characteristics of agents' belief distributions for the macro-economy, with GDP beliefs being the most important factor in the USD market and inflation beliefs being the most important factor in the EUR market.

7 Conclusion

In this paper, we use a comprehensive database of inter-dealer quotes to conduct the first empirical analysis of the dynamics of the swaption cube.

[54] The specification in (19) could also be made more general, but since we are mainly interested in the higher-order moments of the swap rate distributions, we do not pursue such extensions here.

[55] For (31) to be consistent with absence of arbitrage, we need to impose the Feller conditions under $\mathbb{P}$ and $\mathbb{Q}$. As these are typically binding, we often experience a small deterioration in the fit to swaption.

We first analyze the swaption cube from a model independent perspective. We use the fact that for a given swap maturity and option expiry, one can compute conditional moments of the swap rate distribution (under the annuity measure) at a time horizon equal to the option expiry by suitably integrating over swaptions with different strikes. We establish a set of stylized facts regarding the cross-sectional and time-series variation in conditional volatility and skewness of the swap rate distributions. In particular, we show that skewness is stochastic, largely unspanned by rates and volatility, and sometimes changes sign.

We then develop and estimate a stochastic volatility model of the term structure of swap rates that is consistent with these stylized facts. This model is used to infer the conditional swap rate distributions under the risk-neutral measure as well as the physical measure. We show that the risk-neutral swap rate distributions on average exhibit higher volatility and are more skewed towards higher rates than the swap rate distributions under the physical measure.

Finally, we investigate the fundamental drivers of the conditional swap rate distributions. We find that physical volatility and skewness as well as volatility risk premia (defined as the differences between physical and risk-neutral volatility) and skewness risk premia (defined in a similar way) are significantly related to the characteristics of agents' belief distributions for the macro-economy, with GDP beliefs being the most important factor in the USD market, and inflation beliefs being the most important factor in the EUR market. These different market dynamics are consistent with differences in monetary policy objectives in the two economies. The results hold true controlling for other factors that may have an effect on swap rate distributions, including moments of the equity index return distribution, market-wide liquidity, and refinancing activity.

In recent years, a number of equilibrium models for the term structure of interest rates have been proposed.[56] A key challenge for future fixed income research is developing successful equilibrium models for interest rate derivatives. By investigating the fundamental determinants of volatility and skewness of interest rate distributions, our paper provides the first step in this direction.

[56] See, e.g., Piazzesi and Schneider (2007), Bansal and Shaliastovich (2009), Le and Singleton (2010), and Xiong and Yan (2010).

Appendix A

The weights in (17) and (18)

The weights $\zeta_j(t)$ in (17) are given by

$$\zeta_m(t) = \frac{P(t,T_m)}{A(t,T_m,T_n)} \tag{33}$$

$$\zeta_j(t) = -\tau_{j-1}S(t,T_m,T_n)\frac{P(t,T_j)}{A(t,T_m,T_n)}, \quad j = m+1,...,n-1 \tag{34}$$

$$\zeta_n(t) = -(1+\tau_{n-1}S(t,T_m,T_n))\frac{P(t,T_n)}{A(t,T_m,T_n)}, \tag{35}$$

while the weights $\chi_j(t)$ in (18) are given by

$$\chi_j(t) = \frac{\tau_{j-1}P(t,T_j)}{A(t,T_m,T_n)}, \quad j = m+1,...,n. \tag{36}$$

Fourier-based pricing formula for swaptions

The dynamics of the forward swap rate under $\mathbb{A}$ is not entirely affine, due to the stochastic weights $\zeta_j(t)$ and $\chi_j(t)$. However, these are low variance martingales under $\mathbb{A}$, and following much of the literature on LIBOR market models, we may "freeze" these at their initial values to obtain a truly affine model, in which case swaptions can be priced quasi-analytically.[57] First, we find the characteristic function of $S_{m,n}(T_m)$ given by

$$\psi(u,t,T_m,T_n) = E_t^{\mathbb{A}}\left[e^{\mathtt{i}uS_{m,n}(T_m)}\right], \tag{37}$$

where $\mathtt{i} = \sqrt{-1}$. This has an exponentially affine solution as demonstrated in the following proposition:

Proposition 1 *(37) is given by*

$$\psi(u,t,T_m,T_n) = e^{M(T_m-t)+\sum_{j=1}^{2}N_j(T_m-t)v_j(t)+\mathtt{i}uS_{m,n}(t)}, \tag{38}$$

[57] In a LIBOR market model setting, this "freezing" technique results in very small biases in swaptions prices. Extensive simulations show that the biases are also very small in our context (these results are available upon request).

where $M(\tau)$, $N_1(\tau)$, and $N_2(\tau)$ solve the following system of ODEs

$$\frac{dM(\tau)}{d\tau} = N_1(\tau)\eta_1 + N_2(\tau)\eta_2 \tag{39}$$

$$\begin{aligned}\frac{dN_1(\tau)}{d\tau} &= \left(-\kappa_1^{\mathbb{A}} + \mathtt{i}u\sigma_{v1}\sum_{i=1}^{N}\rho_i\sigma_{S,i}(t,T_m,T_n)\right)N_1(\tau) - \kappa_{21}N_2(\tau) + \frac{1}{2}N_1(\tau)^2\sigma_{v1}^2 \\ &\quad -\frac{1}{2}u^2\sum_{i=1}^{N}\sigma_{S,i}(t,T_m,T_n)^2\end{aligned} \tag{40}$$

$$\begin{aligned}\frac{dN_2(\tau)}{d\tau} &= \left(-\kappa_2^{\mathbb{A}} + \mathtt{i}u\sigma_{v2}\sum_{i=1}^{N}\overline{\rho}_i\sigma_{S,i}(t,T_m,T_n)\right)N_2(\tau) - \kappa_{12}N_1(\tau) + \frac{1}{2}N_2(\tau)^2\sigma_{v2}^2 \\ &\quad -\frac{1}{2}u^2\sum_{i=1}^{N}\sigma_{S,i}(t,T_m,T_n)^2\end{aligned} \tag{41}$$

subject to the boundary conditions $M(0) = 0$, $N_1(0) = 0$, and $N_2(0) = 0$.

Proof: Available upon request. ▮

Next, we follow the general approach of Carr and Madan (1999) and Lee (2004) to price swaptions. The idea is that the Fourier transform of the modified swaption price,

$$\widehat{\mathcal{P}}_{m,n}(t,K) = e^{\alpha K}\mathcal{P}_{m,n}(t,K), \tag{42}$$

can be expressed in terms of the characteristic function of $S_{m,n}(T_m)$.[58] The swaption price is then obtained by applying the Fourier inversion theorem. The result is given in the following proposition:

Proposition 2 *The time-t price of a European payer swaption expiring at T_m with strike K on a swap for the period T_m to T_n, $\mathcal{P}_{m,n}(t,K)$, is given by*

$$\mathcal{P}_{m,n}(t,K) = A_{m,n}(t)\frac{e^{-\alpha K}}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-\mathtt{i}uK}\psi(u-\mathtt{i}\alpha,t,T_m,T_n)}{(\alpha+\mathtt{i}u)^2}\right]du. \tag{43}$$

Proof: Available upon request. ▮

[58]The control parameter α must be chosen to ensure that the modified swaption price is L^2 integrable, which is a sufficient condition for its Fourier transform to exist.

Appendix B. Maximum likelihood estimation

The state space form

We cast the model in state space form, which consists of a measurement equation and a transition equation. The measurement equation describes the relationship between the state variables and the prices of swaps and swaptions, while the transition equation describes the discrete-time dynamics of the state variables.

Let X_t denote the vector of state variables. While the transition density of X_t is unknown, its conditional mean and variance is known in closed form, since X_t follows an affine diffusion process. We approximate the transition density with a Gaussian density with identical first and second moments, in which case the transition equation becomes

$$X_t = \Phi_0 + \Phi_X X_{t-1} + w_t, \quad w_t \sim N(0, Q_t), \tag{44}$$

where $Q_t = Q_0 + Q_{v1} v_{1,t} + Q_{v2} v_{2,t}$ and Φ_0, Φ_X, Q_0, Q_{v1}, and Q_{v2} are given in closed form.[59]

The measurement equation is given by

$$Z_t = h(X_t) + u_t, \quad u_t \sim N(0, \Omega), \tag{45}$$

where Z_t is a vector consisting of all the swaptions and underlying swap rates in the time-t swaption cube, h is the pricing function, and u_t is a vector of iid. Gaussian pricing errors with covariance matrix Ω.

Ideally, we would like to fit the model directly to normal implied volatilities, which are more stable than prices (or log-normal implied volatilities) along the swap maturity, option expiry, moneyness, and time-series dimensions. This is not practical, however, since computing implied volatilities from prices requires a numerical inversion for each swaption, which would add an extra layer of complexity to the estimation procedure. Instead, we fit the model to option prices scaled by their normal vegas (i.e., the sensitivities of the swaption prices to variations in volatilities in the normal pricing model).[60]

[59] Approximating the true transition density with a Gaussian, makes this a QML procedure. While QML estimation has been shown to be consistent in many settings, it is in fact not consistent in a Kalman filter setting, since the conditional covariance matrix Q_t in the recursions depends on the Kalman filter estimates of the volatility state variables rather than the true, but unobservable, values; see, e.g., Duan and Simonato (1999). However, simulation results in several papers have shown this issue to be negligible in practice.

[60] This essentially converts swaption pricing errors in terms of prices into swaption pricing errors in terms of normal implied volatilities, via a linear approximation.

To reduce the number of parameters in Ω, we assume that the measurement errors are cross-sectionally uncorrelated (that is, Ω is diagonal), and that one variance, σ^2_{rates}, applies to all pricing errors for swap rates, and that another variance, $\sigma^2_{swaption}$, applies to all pricing errors for scaled swaption prices.

The unscented Kalman filter

If the pricing function were linear, $h(X_t) = h_0 + HX_t$, the Kalman filter would provide efficient estimates of the conditional mean and variance of the state vector. Let $\hat{X}_{t|t-1} = E_{t-1}[X_t]$ and $\hat{Z}_{t|t-1} = E_{t-1}[Z_t]$ denote the expectation of X_t and Z_t, respectively, using information up to and including time $t-1$, and let $P_{t|t-1}$ and $F_{t|t-1}$ denote the corresponding error covariance matrices. Furthermore, let $\hat{X}_t = E_t[X_t]$ denote the expectation of X_t including information at time t, and let P_t denote the corresponding error covariance matrix. The Kalman filter consists of two steps: prediction and update. In the prediction step, $\hat{X}_{t|t-1}$ and $P_{t|t-1}$ are given by

$$\hat{X}_{t|t-1} = \Phi_0 + \Phi_X \hat{X}_{t-1} \tag{46}$$

$$P_{t|t-1} = \Phi_X P_{t-1} \Phi_X' + Q_t, \tag{47}$$

and $\hat{Z}_{t|t-1}$ and $F_{t|t-1}$ are in turn given by

$$\hat{Z}_{t|t-1} = h(\hat{X}_{t|t-1}) \tag{48}$$

$$F_{t|t-1} = HP_{t|t-1}H' + \Omega. \tag{49}$$

In the update step, the state estimate is refined based on the difference between predicted and observed swaps and swaptions, with $\hat{X}_t$ and P_t given by

$$\hat{X}_t = \hat{X}_{t|t-1} + W_t(Z_t - \hat{Z}_{t|t-1}) \tag{50}$$

$$P_t = P_{t|t-1} - W_t F_{t|t-1} W_t', \tag{51}$$

where

$$W_t = P_{t|t-1} H' F^{-1}_{t|t-1} \tag{52}$$

is the covariance between pricing and filtering errors.

In our setting, the pricing function is non-linear for both swaps and swaptions, and the Kalman filter has to be modified. Non-linear state space systems have traditionally been handled with the extended Kalman filter, which effectively linearizes the measure equation around the predicted state. However, in recent years the unscented Kalman filter has emerged as an attractive alternative. Rather than approximating the measurement equation, it uses the true non-linear measurement equation and instead approximates the distribution of the state vector with a deterministically chosen set of sample points, called "sigma points", that completely capture the true mean and covariance of the state vector. When propagated through the non-linear pricing function, the sigma points capture the mean and covariance of swaps and swaptions accurately to the 2nd order (3rd order for true Gaussian states) for any nonlinearity.[61]

More specifically, a set of $2L+1$ sigma points and associated weights are selected according to the following scheme

$$\begin{array}{lclclcll} \hat{\mathcal{X}}^0_{t|t-1} & = & \hat{X}_{t|t-1} & & w^0 & = & \frac{\kappa}{L+\kappa} & \\ \hat{\mathcal{X}}^i_{t|t-1} & = & \hat{X}_{t|t-1} + \left(\sqrt{(L+\kappa)P_{t|t-1}}\right)_i & & w^i & = & \frac{1}{2(L+\kappa)} & i = 1, ..., L \\ \hat{\mathcal{X}}^i_{t|t-1} & = & \hat{X}_{t|t-1} - \left(\sqrt{(L+\kappa)P_{t|t-1}}\right)_i & & w^i & = & \frac{1}{2(L+\kappa)} & i = L+1, ..., 2L, \end{array} \tag{53}$$

where L is the dimension of $\hat{X}_{t|t-1}$, κ is a scaling parameter, w^i is the weight associated with the i'th sigma-point, and $\left(\sqrt{(L+\kappa)P_{t|t-1}}\right)_i$ is the i'th column of the matrix square root. Then, in the prediction step, (48) and (49) are replaced by

$$\hat{Z}_{t|t-1} = \sum_{i=0}^{2L} w^i h(\hat{\mathcal{X}}^i_{t|t-1}) \tag{54}$$

$$F_{t|t-1} = \sum_{i=0}^{2L} w^i (h(\hat{\mathcal{X}}^i_{t|t-1}) - \hat{Z}_{t|t-1})(h(\hat{\mathcal{X}}^i_{t|t-1}) - \hat{Z}_{t|t-1})' + \Omega. \tag{55}$$

The update step is still given by (50) and (51), but with W_t computed as

$$W_t = \sum_{i=0}^{2L} w^i (\hat{\mathcal{X}}^i_{t|t-1} - \hat{X}_{t|t-1})(h(\hat{\mathcal{X}}^i_{t|t-1}) - \hat{Z}_{t|t-1})' F^{-1}_{t|t-1}. \tag{56}$$

Finally, the log-likelihood function is given by

$$\log L = -\frac{1}{2}\log 2\pi \sum_{i=1}^{T} N_t - \frac{1}{2}\sum_{i=1}^{T}\log|F_{t|t-1}| - \frac{1}{2}\sum_{i=1}^{T}(Z_t - \hat{Z}_{t|t-1})' F^{-1}_{t|t-1}(Z_t - \hat{Z}_{t|t-1}), \tag{57}$$

where T is the number of observation dates, and N_t is the dimension of Z_t.

[61] For comparison, the extended Kalman filter estimates the mean and covariance accurately to the 1st order. Note that the computational costs of the extended Kalman filter and the unscented Kalman filter are of the same order of magnitude.

Tenor	Option expiry							
	1 mth	3 mths	6 mths	9 mths	1 yr	2 yrs	5 yrs	10 yrs
				Panel A: USD market				
2 yrs	110.1	112.0	114.4	117.4	120.7	123.6	116.9	98.2
	(35.5)	(31.5)	(27.9)	(26.9)	(26.6)	(24.6)	(18.0)	(12.0)
5 yrs	122.4	122.2	121.8	121.1	121.3	120.4	111.7	93.2
	(37.4)	(33.5)	(29.4)	(27.4)	(26.3)	(23.2)	(16.5)	(10.4)
10 yrs	115.9	115.8	115.4	114.7	114.4	113.3	104.7	86.8
	(36.6)	(32.7)	(28.7)	(26.5)	(25.0)	(22.0)	(15.0)	(9.2)
20 yrs	106.9	105.7	104.0	102.7	101.8	99.5	89.9	74.1
	(36.1)	(31.4)	(26.7)	(24.0)	(21.8)	(18.6)	(12.3)	(7.9)
30 yrs	103.5	101.7	99.9	98.6	97.5	95.1	85.4	69.6
	(37.6)	(31.5)	(26.3)	(23.4)	(20.9)	(17.1)	(10.8)	(6.3)
				Panel B: EUR market				
2 yrs	80.7	81.2	81.6	81.3	80.8	80.6	78.0	72.1
	(29.2)	(24.6)	(20.2)	(17.2)	(15.3)	(12.9)	(8.7)	(6.3)
5 yrs	83.6	82.4	80.9	79.6	78.6	77.1	74.2	69.1
	(25.3)	(21.1)	(17.1)	(14.8)	(13.4)	(11.5)	(8.5)	(7.2)
10 yrs	74.7	74.7	74.3	73.7	73.3	73.4	71.9	67.1
	(23.1)	(20.5)	(17.6)	(15.9)	(15.0)	(13.6)	(9.9)	(7.6)
20 yrs	72.3	72.0	71.2	70.4	70.0	69.3	67.2	61.9
	(30.0)	(26.4)	(21.8)	(19.3)	(18.0)	(15.4)	(10.9)	(8.2)
30 yrs	71.6	71.2	70.3	69.3	68.7	67.9	65.3	59.8
	(36.8)	(32.5)	(26.7)	(23.0)	(20.8)	(17.6)	(12.9)	(9.2)

Notes: The table shows average conditional volatilities (annualized and in basis points) of the future swap rate distributions under the annuity measure $\mathbb{A}$. Standard deviations of conditional volatilities are in parentheses. In the USD market, each statistic is computed on the basis of 419 weekly observations from December 19, 2001 to January 27, 2010. In the EUR market, each statistic is computed on the basis of 449 weekly observations from June 6, 2001 to January 27, 2010.

Table 1: Volatility (annualized) of swap rate distributions

Tenor	Option expiry							
	1 mth	3 mths	6 mths	9 mths	1 yr	2 yrs	5 yrs	10 yrs
	Panel A: USD market							
2 yrs	0.00 (0.30)	0.20 (0.36)	0.24 (0.43)	0.24 (0.40)	0.27 (0.42)	0.30 (0.41)	0.43 (0.33)	0.42 (0.29)
5 yrs	0.01 (0.20)	0.17 (0.23)	0.19 (0.28)	0.20 (0.25)	0.21 (0.28)	0.23 (0.34)	0.33 (0.35)	0.37 (0.30)
10 yrs	−0.00 (0.17)	0.15 (0.18)	0.16 (0.22)	0.15 (0.20)	0.16 (0.23)	0.18 (0.30)	0.29 (0.34)	0.32 (0.31)
20 yrs	−0.03 (0.13)	0.12 (0.13)	0.13 (0.16)	0.12 (0.16)	0.13 (0.19)	0.18 (0.25)	0.27 (0.32)	0.30 (0.36)
30 yrs	−0.04 (0.14)	0.10 (0.13)	0.12 (0.15)	0.11 (0.15)	0.12 (0.17)	0.15 (0.23)	0.22 (0.30)	0.27 (0.33)
	Panel B: EUR market							
2 yrs	−0.00 (0.15)	0.17 (0.21)	0.27 (0.28)	0.31 (0.27)	0.36 (0.29)	0.47 (0.35)	0.60 (0.33)	0.62 (0.38)
5 yrs	−0.15 (0.38)	−0.03 (0.41)	0.04 (0.40)	0.09 (0.33)	0.12 (0.35)	0.22 (0.30)	0.39 (0.27)	0.46 (0.25)
10 yrs	−0.21 (0.37)	−0.11 (0.39)	−0.06 (0.36)	−0.00 (0.29)	0.02 (0.30)	0.12 (0.25)	0.29 (0.25)	0.36 (0.25)
20 yrs	−0.29 (0.28)	−0.16 (0.27)	−0.11 (0.24)	−0.05 (0.21)	−0.02 (0.22)	0.05 (0.22)	0.24 (0.24)	0.35 (0.27)
30 yrs	−0.32 (0.29)	−0.18 (0.27)	−0.13 (0.25)	−0.07 (0.22)	−0.04 (0.24)	0.00 (0.24)	0.22 (0.28)	0.33 (0.34)

Notes: The table shows average conditional skewness of the future swap rate distributions under the annuity measure $\mathbb{A}$. Standard deviations of conditional skewness are in parentheses. In the USD market, each statistic is computed on the basis of 419 weekly observations from December 19, 2001 to January 27, 2010. In the EUR market, each statistic is computed on the basis of 449 weekly observations from June 6, 2001 to January 27, 2010.

Table 2: Skewness of swap rate distributions

Tenor	Option expiry							
	1 mth	3 mths	6 mths	9 mths	1 yr	2 yrs	5 yrs	10 yrs
	Panel A: USD market							
2 yrs	0.09	0.12	0.16	0.17	0.17	0.15	0.12	0.11
5 yrs	0.10	0.10	0.12	0.17	0.19	0.17	0.16	0.16
10 yrs	0.08	0.07	0.09	0.16	0.20	0.17	0.20	0.25
20 yrs	0.07	0.05	0.07	0.10	0.15	0.16	0.19	0.28
30 yrs	0.06	0.04	0.03	0.06	0.12	0.19	0.22	0.24
	Panel B: EUR market							
2 yrs	0.08	0.11	0.14	0.18	0.20	0.21	0.21	0.19
5 yrs	0.03	0.05	0.08	0.10	0.14	0.20	0.25	0.29
10 yrs	0.01	0.04	0.07	0.10	0.14	0.22	0.29	0.34
20 yrs	0.01	0.05	0.08	0.11	0.15	0.23	0.29	0.34
30 yrs	0.02	0.05	0.09	0.12	0.15	0.22	0.26	0.30

Notes: The table shows the R^2s from regressing weekly changes in conditional skewness of the swap rate distributions on the main principal components (PCs) of weekly changes in all forward swap rates and the main PCs of weekly changes in conditional variances of all swap rate distributions. We also include the squared PCs in the regressions. In the USD market, the time-series is from December 19, 2001 to January 27, 2010. In the EUR market, the time-series is from June 6, 2001 to January 27, 2010.

Table 3: Evidence for unspanned stochastic skewness

	USD market		*EUR market*	
	SV1	SV2	SV1	SV2
α_1	0.0072 (0.0001)	0.0065 (0.0001)	0.0090 (0.0001)	0.0069 (0.0001)
α_2	0.0021 (0.0000)	0.0028 (0.0000)	0.0016 (0.0000)	0.0061 (0.0001)
α_3	0.0049 (0.0001)	0.0045 (0.0001)	0.0020 (0.0000)	0.0022 (0.0000)
ξ	0.0104 (0.0002)	0.0086 (0.0001)	0.0115 (0.0001)	0.0005 (0.0000)
γ	0.2471 (0.0046)	0.2435 (0.0034)	0.2795 (0.0042)	0.1670 (0.0023)
φ	0.0479 (0.0018)	0.0464 (0.0018)	0.0435 (0.0018)	0.0441 (0.0017)
ρ_1	0.2546 (0.0202)	-0.1843 (0.0270)	0.0887 (0.0188)	-0.0446 (0.0265)
ρ_2	0.5752 (0.0199)	-0.0945 (0.0298)	0.5263 (0.0247)	0.3736 (0.0207)
ρ_3	-0.0019 (0.0204)	-0.3208 (0.0207)	-0.0509 (0.0201)	-0.5463 (0.0211)
$\overline{\rho}_1$		0.4753 (0.0230)		0.3736 (0.0216)
$\overline{\rho}_2$		0.5748 (0.0229)		0.5582 (0.0178)
$\overline{\rho}_3$		0.5247 (0.0201)		0.5199 (0.0264)
$\kappa_1 = \kappa_2$	0.5045 (0.0066)	0.4156 (0.0067)	0.5300 (0.0072)	0.6948 (0.0109)
$\eta_1 = \eta_2$	0.4957 (0.0075)	0.2404 (0.0029)	0.2797 (0.0039)	0.1981 (0.0022)
ν	-0.2169 (0.0823)	-0.0873 (0.0225)	-0.2014 (0.0681)	-0.1320 (0.0427)
$\overline{\nu}$		-0.2992 (0.0909)		-0.4377 (0.1329)
$\sigma_{rates} \times 10^4$	6.3465 (0.0714)	6.3676 (0.0751)	5.4623 (0.0826)	5.4587 (0.1001)
$\sigma_{swaptions} \times 10^4$	5.5125 (0.0775)	4.5998 (0.0861)	4.8212 (0.0540)	4.3356 (0.0550)
Log-likelihood $\times 10^4$	-27.1280	-25.9420	-23.6951	-23.2632

Notes: Maximum-likelihood estimates of the SV1 and SV2 specifications. The sample period is December 19, 2001 to January 27, 2010 in the USD market and June 6, 2001 to January 27, 2010 in the EUR market. Outer-product standard errors are in parentheses. σ_{rates} denotes the standard deviation of swap rate measurement errors and $\sigma_{swaptions}$ denotes the standard deviation of scaled swaption price measurement errors. For identification purposes, we set $\sigma_{v1} = 1$ in SV1, and $\sigma_{v1} = \sigma_{v2} = 1$ in SV2.

Table 4: Parameter estimates

	USD market			*EUR market*		
	SV1	SV2	SV2-SV1	SV1	SV2	SV2-SV1
Swaption IV	5.44	4.58	-0.87^{***} (−6.04)	4.72	4.31	-0.41^{***} (−5.37)
Volatility	4.96	4.83	-0.13^{**} (−1.97)	3.83	3.76	-0.07^{*} (−1.88)
Skewness	0.21	0.05	-0.16^{***} (−6.71)	0.24	0.08	-0.17^{***} (−10.15)
Kurtosis	0.34	0.21	-0.13^{*} (−1.81)	0.47	0.30	-0.17^{***} (−2.64)

Notes: The table compares the SV1 and SV2 specifications in terms of their ability to match the normal implied volatilities (in basis points) as well as conditional volatility (annualized and in basis points), skewness, and kurtosis of the future swap rate distributions under the annuity measure $\mathbb{A}$. It reports means of RMSE time series of implied volatilities and swap rate moments. It also reports mean differences in RMSEs between the two model specifications. T-statistics, corrected for serial correlation up to 26 lags (i.e., two quarters), are in parentheses. In the USD market, each statistic is computed on the basis of 419 weekly observations from December 19, 2001 to January 27, 2010. In the EUR market, each statistic is computed on the basis of 449 weekly observations from June 6, 2001 to January 27, 2010. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

Table 5: Overall comparison between models

Tenor	Option expiry							
	1 mth	3 mths	6 mths	9 mths	1 yr	2 yrs	5 yrs	10 yrs
				Panel A: USD market				
2 yrs	-0.08^{***}	-0.12^{***}	-0.17^{***}	-0.20^{***}	-0.25^{***}	-0.29^{***}	-0.14^{***}	-0.05^{**}
	(-4.44)	(-3.94)	(-4.17)	(-4.56)	(-5.67)	(-6.97)	(-3.29)	(-2.46)
5 yrs	-0.06^{***}	-0.09^{***}	-0.14^{***}	-0.19^{***}	-0.24^{***}	-0.29^{***}	-0.22^{***}	-0.07^{***}
	(-3.45)	(-3.17)	(-3.98)	(-5.17)	(-6.68)	(-7.84)	(-6.47)	(-3.68)
10 yrs	-0.04^{***}	-0.07^{***}	-0.12^{***}	-0.17^{***}	-0.21^{***}	-0.27^{***}	-0.23^{***}	-0.07^{***}
	(-2.69)	(-2.75)	(-3.36)	(-5.19)	(-5.89)	(-6.74)	(-6.53)	(-2.95)
20 yrs	-0.07^{***}	-0.06^{***}	-0.13^{***}	-0.15^{***}	-0.17^{***}	-0.24^{***}	-0.26^{***}	-0.12^{***}
	(-3.79)	(-3.10)	(-4.81)	(-5.27)	(-5.51)	(-6.78)	(-7.67)	(-4.39)
30 yrs	-0.06^{***}	-0.06^{**}	-0.10^{***}	-0.14^{***}	-0.16^{***}	-0.24^{***}	-0.27^{***}	-0.14^{***}
	(-3.29)	(-2.17)	(-3.32)	(-4.42)	(-4.91)	(-6.49)	(-7.94)	(-4.50)
				Panel B: EUR market				
2 yrs	-0.04^{**}	-0.11^{***}	-0.15^{***}	-0.17^{***}	-0.21^{***}	-0.26^{***}	-0.27^{***}	-0.26^{***}
	(-2.18)	(-4.00)	(-4.11)	(-4.07)	(-4.07)	(-3.84)	(-4.81)	(-6.44)
5 yrs	-0.06^{***}	-0.15^{***}	-0.17^{***}	-0.20^{***}	-0.16^{***}	-0.20^{***}	-0.15^{***}	-0.20^{***}
	(-3.13)	(-7.39)	(-6.93)	(-6.54)	(-4.24)	(-3.93)	(-2.62)	(-5.18)
10 yrs	-0.07^{***}	-0.15^{***}	-0.20^{***}	-0.22^{***}	-0.22^{***}	-0.21^{***}	-0.10^{*}	-0.11^{***}
	(-3.01)	(-5.26)	(-6.17)	(-6.35)	(-5.71)	(-4.72)	(-1.95)	(-3.01)
20 yrs	-0.05	-0.11^{***}	-0.19^{***}	-0.24^{***}	-0.27^{***}	-0.25^{***}	-0.09^{**}	-0.09^{***}
	(-1.45)	(-3.06)	(-4.32)	(-5.41)	(-5.96)	(-5.70)	(-2.45)	(-2.73)
30 yrs	-0.05^{*}	-0.13^{***}	-0.19^{***}	-0.19^{***}	-0.18^{***}	-0.24^{***}	-0.08^{**}	-0.08^{***}
	(-1.68)	(-3.63)	(-4.17)	(-3.77)	(-3.55)	(-5.55)	(-2.28)	(-2.64)

Notes: The table compares the SV1 and SV2 specifications in terms of their ability to match skewness of the future swap rate distributions under the annuity measure $\mathbb{A}$. For each tenor – option expiry category, the table reports the mean differences in absolute skewness errors between the two specifications, where skewness errors are the differences between the model-implied skewness and the model independent skewness. T-statistics, corrected for serial correlation up to 26 lags (i.e., two quarters), are in parentheses. In the USD market, each statistic is computed on the basis of 419 weekly observations from December 19, 2001 to January 27, 2010. In the EUR market, each statistic is computed on the basis of 449 weekly observations from June 6, 2001 to January 27, 2010. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

Table 6: Evaluating fit to skewness

Tenor	Option expiry							
	1 mth	3 mths	6 mths	9 mths	1 yr	2 yrs	5 yrs	10 yrs
				Panel A: USD market				
2 yrs	−0.93	−2.43	−4.13	−5.20	−5.72	−8.24	−14.27	−16.81
	(0.45)	(1.16)	(1.94)	(2.40)	(2.58)	(3.46)	(4.77)	(3.67)
5 yrs	−1.02	−2.67	−4.54	−5.72	−6.27	−8.96	−14.80	−16.66
	(0.49)	(1.27)	(2.13)	(2.64)	(2.84)	(3.77)	(4.93)	(3.56)
10 yrs	−1.00	−2.62	−4.46	−5.62	−6.17	−8.79	−14.30	−15.94
	(0.48)	(1.25)	(2.10)	(2.60)	(2.81)	(3.73)	(4.80)	(3.42)
20 yrs	−0.88	−2.30	−3.95	−4.99	−5.50	−7.94	−13.21	−15.05
	(0.42)	(1.10)	(1.86)	(2.32)	(2.52)	(3.40)	(4.52)	(3.34)
30 yrs	−0.82	−2.16	−3.70	−4.70	−5.19	−7.56	−12.81	−14.87
	(0.40)	(1.03)	(1.75)	(2.19)	(2.38)	(3.25)	(4.43)	(3.37)
				Panel B: EUR market				
2 yrs	−0.56	−1.47	−2.51	−3.14	−3.43	−4.81	−7.77	−9.47
	(0.16)	(0.39)	(0.61)	(0.71)	(0.72)	(0.77)	(0.61)	(0.31)
5 yrs	−0.55	−1.45	−2.47	−3.11	−3.40	−4.80	−7.82	−9.48
	(0.15)	(0.38)	(0.60)	(0.70)	(0.72)	(0.77)	(0.61)	(0.31)
10 yrs	−0.54	−1.41	−2.41	−3.03	−3.33	−4.72	−7.75	−9.40
	(0.15)	(0.37)	(0.59)	(0.69)	(0.70)	(0.76)	(0.60)	(0.30)
20 yrs	−0.50	−1.32	−2.26	−2.85	−3.13	−4.49	−7.55	−9.32
	(0.14)	(0.35)	(0.56)	(0.65)	(0.67)	(0.72)	(0.58)	(0.30)
30 yrs	−0.48	−1.28	−2.19	−2.77	−3.05	−4.41	−7.55	−9.45
	(0.14)	(0.34)	(0.54)	(0.64)	(0.65)	(0.71)	(0.59)	(0.31)

Notes: Volatility risk premia are defined as the differences between conditional volatilities (annualized and in basis points) of the future swap rate distributions under the physical measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$. The table shows averages of volatility risk premia and, in parentheses, standard deviations of volatility risk premia. In the USD market, each statistic is computed on the basis of 419 weekly observations from December 19, 2001 to January 27, 2010. In the EUR market, each statistic is computed on the basis of 449 weekly observations from June 6, 2001 to January 27, 2010.

Table 7: Volatility risk premia

Tenor	Option expiry							
	1 mth	3 mths	6 mths	9 mths	1 yr	2 yrs	5 yrs	10 yrs
	Panel A: USD market							
2 yrs	−0.008	−0.030	−0.074	−0.096	−0.116	−0.222	−0.419	−0.689
	(0.015)	(0.020)	(0.044)	(0.051)	(0.055)	(0.071)	(0.053)	(0.070)
5 yrs	−0.007	−0.031	−0.075	−0.098	−0.118	−0.231	−0.451	−0.712
	(0.012)	(0.020)	(0.045)	(0.052)	(0.057)	(0.076)	(0.055)	(0.067)
10 yrs	−0.007	−0.031	−0.075	−0.098	−0.119	−0.237	−0.473	−0.724
	(0.012)	(0.020)	(0.045)	(0.053)	(0.059)	(0.079)	(0.055)	(0.060)
20 yrs	−0.007	−0.029	−0.073	−0.096	−0.117	−0.237	−0.478	−0.722
	(0.016)	(0.020)	(0.044)	(0.052)	(0.058)	(0.081)	(0.056)	(0.051)
30 yrs	−0.008	−0.029	−0.072	−0.094	−0.115	−0.235	−0.478	−0.723
	(0.019)	(0.019)	(0.043)	(0.051)	(0.057)	(0.080)	(0.056)	(0.046)
	Panel B: EUR market							
2 yrs	−0.008	−0.026	−0.068	−0.093	−0.117	−0.241	−0.407	−0.533
	(0.033)	(0.021)	(0.047)	(0.058)	(0.066)	(0.098)	(0.086)	(0.054)
5 yrs	−0.007	−0.034	−0.086	−0.113	−0.138	−0.266	−0.429	−0.552
	(0.031)	(0.026)	(0.057)	(0.068)	(0.075)	(0.107)	(0.089)	(0.055)
10 yrs	−0.008	−0.039	−0.097	−0.125	−0.151	−0.284	−0.450	−0.567
	(0.036)	(0.029)	(0.062)	(0.073)	(0.080)	(0.111)	(0.090)	(0.054)
20 yrs	−0.006	−0.039	−0.099	−0.128	−0.155	−0.292	−0.463	−0.572
	(0.047)	(0.029)	(0.062)	(0.073)	(0.080)	(0.110)	(0.088)	(0.053)
30 yrs	−0.012	−0.039	−0.097	−0.126	−0.153	−0.291	−0.467	−0.574
	(0.048)	(0.028)	(0.060)	(0.071)	(0.078)	(0.107)	(0.085)	(0.052)

Notes: Skewness risk premia are defined as the differences between conditional skewness of the future swap rate distributions under the physical measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$. The table shows averages of skewness risk premia and, in parentheses, standard deviations of skewness risk premia. In the USD market, each statistic is computed on the basis of 419 weekly observations from December 19, 2001 to January 27, 2010. In the EUR market, each statistic is computed on the basis of 449 weekly observations from June 6, 2001 to January 27, 2010.

Table 8: Skewness risk premia

	$GDPvol$	$GDPskew$	$INFvol$	$INFskew$	$EQvol$	$EQskew$	$ILLIQ$	$REFI$	R^2
vol	21.428^{***}	8.401^{**}	30.536	−6.701					0.404
	(4.888)	(2.064)	(0.854)	(−0.690)					
vol	19.027^{***}	0.892	51.982	9.763	77.317^{***}	127.823^{***}	11.401^{**}	2.142^{**}	0.585
	(6.345)	(0.199)	(1.390)	(1.427)	(4.619)	(3.069)	(2.456)	(2.077)	
$volPrem$	-3.707^{***}	-0.736^{***}	-4.900^{**}	0.496					0.540
	(−9.589)	(−2.628)	(−2.063)	(0.691)					
$volPrem$	-3.397^{***}	−0.288	−2.758	−0.171	-7.050^{***}	-6.225^{*}	-0.897^{**}	-0.162^{*}	0.641
	(−10.613)	(−1.002)	(−1.021)	(−0.292)	(−5.980)	(−1.880)	(−2.015)	(−1.761)	
$skew$	0.290^{*}	0.352^{***}	−0.726	−0.060					0.413
	(1.907)	(2.773)	(−1.139)	(−0.304)					
$skew$	0.199^{*}	0.284^{**}	−0.421	0.123	0.729^{*}	3.971^{***}	−0.192	0.017	0.544
	(1.677)	(2.373)	(−0.615)	(0.985)	(1.719)	(3.174)	(−1.241)	(0.810)	
$skewPrem$	0.023^{*}	0.011^{*}	0.074	-0.035^{*}					0.134
	(1.741)	(1.814)	(1.208)	(−1.786)					
$skewPrem$	0.008^{*}	0.007	−0.004	−0.015	0.045^{*}	-0.203^{*}	0.010	−0.002	0.248
	(1.760)	(1.319)	(−0.088)	(−1.495)	(1.892)	(−1.931)	(1.447)	(−1.301)	

Notes: The table reports estimates of the MIDAS regression specification (29) in which the cross-sectional average of USD physical volatility (vol), volatility risk premia ($volPrem$), physical skewness ($skew$), or skewness risk premia ($skewPrem$) is regressed on a constant, dispersion and skewness of agents' belief distributions for future U.S. real GDP growth and inflation ($GDPvol$, $GDPskew$, $INFvol$, and $INFskew$), volatility and skewness of the risk-neutral S&P 500 index return distribution ($EQvol$ and $EQskew$), the spread between the 3-month OIS rate and the 3-month Treasury bill yield ($ILLIQ$), and the MBA Refinancing Index ($REFI$). Physical volatility and volatility risk premia are measured in basis points, and the MBA Refinancing Index is divided by 1000. Estimation is by non-linear least squares. T-statistics, corrected for heteroscedasticity and serial correlation up to 26 lags (i.e., two quarters), are in parentheses. The sample period is December 19, 2001 to January 27, 2010. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

Table 9: Fundamental drivers of USD swap rate distributions

	$GDPvol$	$GDPskew$	$INFvol$	$INFskew$	$EQvol$	$EQskew$	$ILLIQ$	$REFI$	R^2
vol	-2.193	3.153^{*}	106.612^{***}	10.677^{**}					0.630
	(-0.602)	(1.759)	(6.296)	(2.183)					
vol	2.730	2.176^{*}	60.941^{***}	4.057	25.633^{***}	3.357	4.232^{*}	0.409	0.698
	(1.092)	(1.856)	(4.924)	(1.017)	(3.817)	(0.174)	(1.882)	(0.749)	
$volPrem$	-0.781^{**}	-0.447^{*}	-7.063^{***}	-0.254					0.411
	(-2.063)	(-1.660)	(-3.937)	(-0.364)					
$volPrem$	-0.704^{**}	-0.425	-4.759^{***}	-0.337	-2.665^{***}	-4.165	-0.520	-0.067	0.434
	(-2.300)	(-1.588)	(-3.160)	(-0.492)	(-2.791)	(-1.426)	(-1.587)	(-0.884)	
$skew$	0.256^{**}	0.047	-1.071^{**}	0.483^{**}					0.269
	(2.468)	(0.468)	(-2.141)	(2.548)					
$skew$	0.114	0.126	-0.877^{*}	0.351^{*}	-0.767^{*}	2.911^{***}	0.134	0.009	0.462
	(1.226)	(1.338)	(-1.718)	(1.846)	(-1.951)	(2.780)	(1.266)	(0.351)	
$skewPrem$	-0.044^{*}	0.003	0.237^{**}	-0.098^{***}					0.228
	(-1.847)	(0.158)	(2.248)	(-2.638)					
$skewPrem$	-0.035^{*}	-0.011	0.191^{*}	-0.071^{**}	0.159^{**}	-0.396^{*}	-0.037	-0.002	0.329
	(-1.733)	(-0.584)	(1.715)	(-1.971)	(2.080)	(-1.806)	(-1.577)	(-0.376)	

Notes: The table reports estimates of the MIDAS regression specification (29) in which the cross-sectional average of EUR physical volatility (vol), volatility risk premia ($volPrem$), physical skewness ($skew$), or skewness risk premia ($skewPrem$) is regressed on a constant, dispersion and skewness of agents' belief distributions for future Eurozone real GDP growth and inflation ($GDPvol$, $GDPskew$, $INFvol$, and $INFskew$), volatility and skewness of the risk-neutral Eurostoxx 50 index return distribution ($EQvol$ and $EQskew$), the spread between the 3-month OIS rate and the 3-month German Bubill yield ($ILLIQ$), and the MBA Refinancing Index ($REFI$). Physical volatility and volatility risk premia are measured in basis points, and the MBA Refinancing Index is divided by 1000. Estimation is by non-linear least squares. T-statistics, corrected for heteroscedasticity and serial correlation up to 26 lags (i.e., two quarters), are in parentheses. The sample period is June 6, 2001 to January 27, 2010. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

Table 10: Fundamental drivers of EUR swap rate distributions

	$\Delta GDPvol$	$\Delta GDPskew$	$\Delta INFvol$	$\Delta INFskew$	$\Delta EQvol$	$\Delta EQskew$	$\Delta ILLIQ$	$\Delta REFI$	R^2
Δvol	23.591^{**} (2.343)	-0.474 (-0.083)	8.335 (0.221)	8.059 (1.175)					0.176
Δvol	21.784^{**} (2.139)	1.790 (0.244)	12.096 (0.322)	11.926 (1.513)	18.723 (0.556)	-94.012 (-0.944)	12.153^{**} (2.013)	2.154 (1.103)	0.311
$\Delta volPrem$	-1.742^{*} (-1.692)	-0.741 (-1.267)	-0.210 (-0.055)	-1.049 (-1.495)					0.187
$\Delta volPrem$	-2.125^{*} (-1.885)	-0.803 (-0.989)	-1.842 (-0.443)	-0.635 (-0.728)	-0.203 (-0.055)	17.462 (1.585)	0.082 (0.122)	0.033 (0.153)	0.280
$\Delta skew$	-0.077 (-0.269)	0.436^{***} (2.672)	1.483 (1.380)	-0.117 (-0.595)					0.258
$\Delta skew$	-0.041 (-0.151)	0.433^{**} (2.196)	1.931^{*} (1.910)	-0.282 (-1.331)	0.585 (0.646)	-0.337 (-0.126)	-0.414^{**} (-2.548)	0.045 (0.853)	0.502
$\Delta skewPrem$	0.018 (0.593)	-0.013 (-0.765)	-0.181 (-1.569)	0.038^{*} (1.817)					0.141
$\Delta skewPrem$	0.039 (1.541)	-0.002 (-0.110)	-0.151 (-1.609)	0.037^{*} (1.852)	-0.146^{*} (-1.724)	-1.021^{***} (-4.095)	0.023 (1.497)	0.003 (0.669)	0.472

Notes: The table reports estimates of the regression specification (32) in which the quarterly change in the cross-sectional average of USD physical volatility (vol), volatility risk premia ($volPrem$), physical skewness ($skew$), or skewness risk premia ($skewPrem$) is regressed on a constant and the quarterly changes in the dispersion and skewness of agents' belief distributions for future U.S. real GDP growth and inflation ($GDPvol$, $GDPskew$, $INFvol$, and $INFskew$), the volatility and skewness of the risk-neutral S&P 500 index return distribution ($EQvol$ and $EQskew$), the spread between the 3-month OIS rate and the 3-month Treasury bill yield ($ILLIQ$), and the MBA Refinancing Index ($REFI$). Physical volatility and volatility risk premia are measured in basis points, and the MBA Refinancing Index is divided by 1000. Estimation is by ordinary least squares. T-statistics, corrected for heteroscedasticity and serial correlation up to 2 lags (i.e., two quarters), are in parentheses. The sample period is January, 2002 to January, 2010. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

Table 11: Fundamental drivers of USD swap rate distributions, regression in differences

	$\Delta GDPvol$	$\Delta GDPskew$	$\Delta INFvol$	$\Delta INFskew$	$\Delta EQvol$	$\Delta EQskew$	$\Delta ILLIQ$	$\Delta REFI$	R^2
Δvol	-0.703	5.926^{*}	66.321^{**}	-1.567					0.169
	(-0.146)	(1.883)	(2.122)	(-0.249)					
Δvol	-1.506	6.155^{*}	67.924^{**}	-4.409	34.517^{*}	-43.332	5.718	-1.282	0.308
	(-0.305)	(1.744)	(2.055)	(-0.676)	(1.737)	(-0.807)	(1.597)	(-1.108)	
$\Delta volPrem$	-0.317^{*}	-0.206^{*}	-2.516^{**}	0.250					0.226
	(-1.676)	(-1.671)	(-2.055)	(1.013)					
$\Delta volPrem$	-0.350^{*}	-0.265^{*}	-2.940^{**}	0.281	-0.577	2.546	-0.016	0.036	0.285
	(-1.714)	(-1.818)	(-2.155)	(1.043)	(-0.703)	(1.148)	(-0.105)	(0.762)	
$\Delta skew$	0.193^{*}	0.055	0.984	0.294^{**}					0.257
	(1.776)	(0.769)	(1.398)	(2.073)					
$\Delta skew$	0.224^{**}	0.070	1.052	0.297^{**}	-0.867^{*}	1.056	0.083	0.025	0.357
	(1.975)	(0.863)	(1.386)	(1.981)	(-1.901)	(0.855)	(1.008)	(0.932)	
$\Delta skewPrem$	-0.051^{*}	0.003	-0.202	-0.061^{*}					0.309
	(-1.882)	(0.164)	(-1.158)	(-1.732)					
$\Delta skewPrem$	-0.052^{*}	-0.003	-0.205	-0.063^{*}	0.242^{**}	0.108	-0.027	-0.006	0.439
	(-1.891)	(-0.161)	(-1.124)	(-1.763)	(2.204)	(0.363)	(-1.346)	(-0.924)	

Notes: The table reports estimates of the regression specification (32) in which the quarterly change in the cross-sectional average of EUR physical volatility (vol), volatility risk premia ($volPrem$), physical skewness ($skew$), or skewness risk premia ($skewPrem$) is regressed on a constant and the quarterly changes in the dispersion and skewness of agents' belief distributions for future Eurozone real GDP growth and inflation ($GDPvol$, $GDPskew$, $INFvol$, and $INFskew$), the volatility and skewness of the risk-neutral Eurostoxx 50 index return distribution ($EQvol$ and $EQskew$), the spread between the 3-month OIS rate and the 3-month German Bubill yield ($ILLIQ$), and the MBA Refinancing Index ($REFI$). Physical volatility and volatility risk premia are measured in basis points, and the MBA Refinancing Index is divided by 1000. Estimation is by ordinary least squares. T-statistics, corrected for heteroscedasticity and serial correlation up to 2 lags (i.e., two quarters), are in parentheses. The sample period is July, 2001 to January, 2010. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

Table 12: Fundamental drivers of EUR swap rate distributions, regression in differences

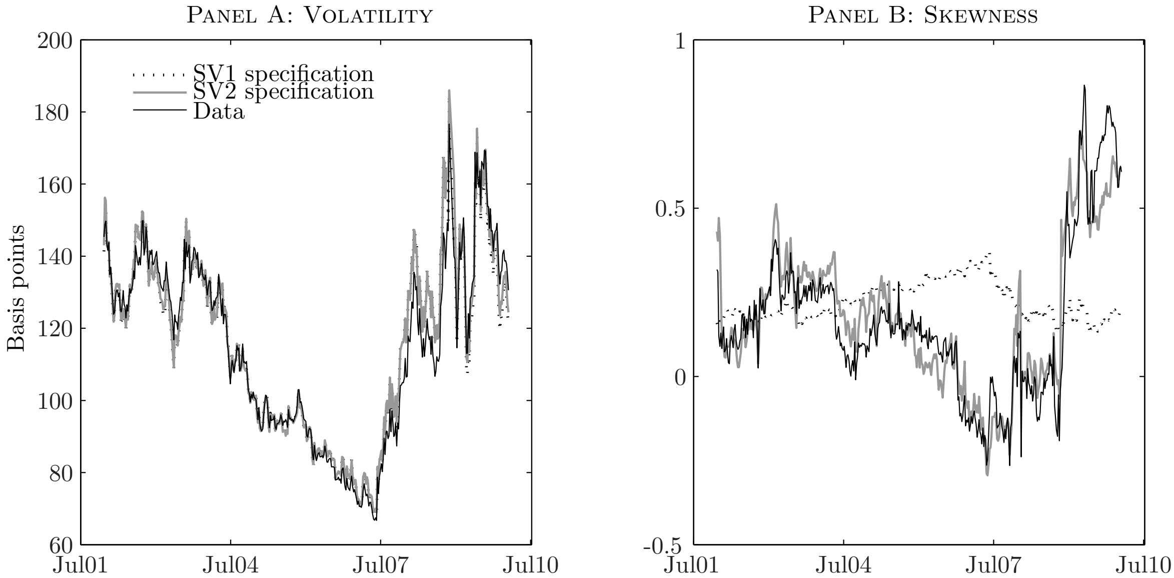

Figure 1: Time-series of volatility and skewness of the conditional 1-year ahead distribution of the USD 10-year swap rate

Notes: Panel A displays conditional volatility, measured in basis points, and Panel B displays conditional skewness. The moments are computed under the annuity measure $\mathbb{A}$. The time-series consist of 419 weekly observations from December 19th, 2001 to January 27th, 2010.

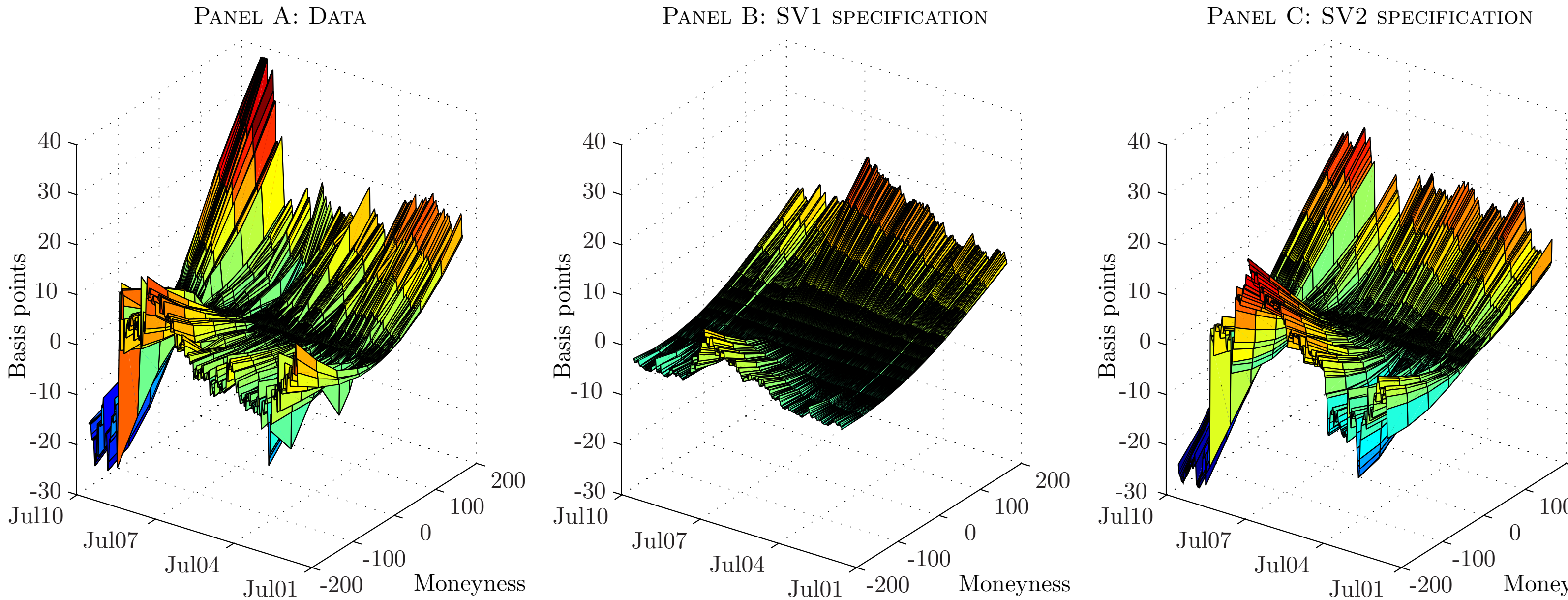

Figure 2: Time-series of the USD normal implied volatility smile of the 1-year option on 10-year swap rate

Notes: Panel A displays the data, Panel B displays the smiles obtained in the SV1 specification, and Panel C displays the smiles obtained in the SV2 specification. The smiles are the differences between implied volatilities for different strikes and ATM implied volatilities. Implied volatilities are measured in basis points. The time-series consist of 419 weekly observations from December 19th, 2001 to January 27th, 2010.

References

ANDERSEN, L. AND V. PITERBARG (2010): *Interest Rate Modeling,* Atlantic Financial Press.

ANDERSEN, T. G. AND L. BENZONI (2010): "Do bonds span volatility risk in the U.S. Treasury market? A specification test for affine term structure models," *Journal of Finance*, 65:603–653.

ANG, A. AND M. PIAZZESI (2003): "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables," *Journal of Monetary Economics*, 50:745–787.

ANG, A., M. PIAZZESI, AND M. WEI (2006): "What does the yield curve tell us about GDP growth?," *Journal of Econometrics*, 131:359–403.

BAKSHI, G., N. KAPADIA, AND D. MADAN (2003): "Stock return characteristics, skew laws, and the differential pricing of individual equity options," *Review of Financial Studies*, 16:101–143.

BAKSHI, G. AND D. MADAN (2000): "Spanning and derivative-security valuation," *Journal of Financial Economics*, 55:205–238.

BANSAL, R. AND I. SHALIASTOVICH (2009): "A long-run risks explanation of predictability puzzles in bond and currency markets," Working paper, Duke University.

BARCLAYS (2010): *2010 US Volatility Outlook,*

BEBER, A. AND M. BRANDT (2006): "The effects of macroeconomic news on beliefs and preferences: Evidence from the options market," *Journal of Monetary Economics*, pages 1997–2039.

BEKAERT, G., S. CHO, AND A. MORENO (2010): "New-Keynesian macroeconomics and the term structure," *Journal of Money, Credit and Banking*, 42:33–62.

BEKAERT, G. AND E. ENGSTROM (2009): "Asset return dynamics under bad environment-good environment fundamentals," Working paper, Columbia Business School.

BIKBOV, R. AND M. CHERNOV (2010): "No-arbitrage macroeconomic determinants of the yield curve," *Journal of Econometrics*, 159:166–182.

BIS (2010): *OTC derivatives market activity in the second half of 2009,*

BLACK, F. (1976): "The pricing of commodity contracts," *Journal of Financial Economics*, 3:167–179.

BOLLEN, N. AND R. WHALEY (2004): "Does net buying pressure affect the shape of implied volatility functions?," *Journal of Finance*, 59:711–753.

BOLLERSLEV, T., N. SIZOVA, AND G. TAUCHEN (2009): "Volatility in equilibrium: Asymmetries and dynamic dependencies," Working paper, Duke University.

BOLLERSLEV, T., G. TAUCHEN, AND H. ZHOU (2009): "Expected stock returns and variance risk premia," *Review of Financial Studies*, 22:4463–4492.

BONGAERTS, D., F. DE JONG, AND J. DRIESSEN (2010): "Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market," Working paper, Tilburg University, forthcoming *Journal of Finance*.

BREEDEN, D. AND R. LITZENBERGER (1978): "Prices of state-contingent claims implicit in option prices," *Journal of Business*, pages 621–651.

BURASCHI, A. AND A. JILTSOV (2006): "Model uncertainty and option markets with heterogeneous beliefs," *Journal of Finance*, 61:2841–2897.

BURASCHI, A., F. TROJANI, AND A. VEDOLIN (2009): "When uncertainty blows in the orchard: Co-movement and equilibrium volatility risk premia," Working paper, Imperial College and University of Lugano.

CARR, P., X. GARBAIX, AND L. WU (2009): "Linearity-generating processes, unspanned stochastic volatility, and interest-rate option pricing," Working paper, New York University.

CARR, P. AND D. MADAN (1999): "Option valuation using the Fast Fourier Transform," *Journal of Computational Finance*, 2:61–73.

——— (2001): "Optimal positioning in derivatives securities," *Quantitative Finance*, 1:19–37.

CARR, P. AND L. WU (2007): "Stochastic skew in currency options," *Journal of Financial Economics*, 86:213–247.

CETIN, U., R. JARROW, P. PROTTER, AND M. WARACHKA (2006): "Pricing options in an extended Black Scholes economy with illiquidity: Theory and empirical evidence," *Review of Financial Studies*, 19:493–529.

CHEREDITO, P., D. FILIPOVIC, AND R. KIMMEL (2007): "Market price of risk specifications for affine models: Theory and evidence," *Journal of Financial Economics*, 83:123–170.

CHRISTOFFERSEN, P., S. HESTON, AND K. JAKOBS (2009): "The shape and term structure of the index option smirk: why multifactor stochastic volatility models work so well," *Management Science*, 55:1914–1932.

CHRISTOFFERSEN, P., K. JAKOBS, L. KAROUI, AND K. MIMOUNI (2009): "Non-linear filtering in affine term structure models: Evidence from the term structure of swap rates," Working paper, McGill University.

CHUN, A. L. (2010): "Expectations, bond yields, and monetary policy," Working paper, Copenhagen Business School, forthcoming *Review of Financial Studies*.

CIESLAK, A. AND P. POVALA (2010): "Understanding the term structure of yield curve volatility," Working paper, University of Lugano.

CLARIDA, R., J. GALI, AND M. GERTLER (2000): "Monetary policy rules and macroeconomic stability: Evidence and some theory," *The Quarterly Journal of Economics*, 115:147–180.

COLLIN-DUFRESNE, P. AND R. GOLDSTEIN (2002): "Do bonds span the fixed income markets? Theory and evidence for unspanned stochastic volatility," *Journal of Finance*, 57:1685–1730.

COLLIN-DUFRESNE, P., R. GOLDSTEIN, AND C. JONES (2009): "Can interest rate volatility be extracted from the cross section of bond yields?," *Journal of Financial Economics*, 94:47–66.

DAS, S. R. AND R. K. SUNDARAM (1999): "Of smiles and smirks: A term structure perspective," *Journal of Financial and Quantitative Analysis*, 34:211–239.

DAVID, A. AND P. VERONESI (2002): "Options prices with uncertain fundamentals," Working paper, University of Chicago.

——— (2009): "Macroeconomic uncertainty and fear measures extracted from index options," Working paper, University of Chicago.

DE JONG, F., J. DRIESSEN, AND A. PELSSER (2004): "On the information in the interest rate term structure and option prices," *Review of Derivatives Research*, 7:99–127.

DEUSKAR, P., A. GUPTA, AND M. SUBRAHMANYAM (2008): "The economic determinants of interest rate option smiles," *Journal of Banking and Finance*, 32:714–728.

——— (2010): "Liquidity effects in OTC options markets: Premium or discount?," Working paper, New York University, forthcoming *Journal of Financial Markets*.

DRECHSLER, I. AND A. YARON (2009): "What's vol got to do with it," Working paper, NYU Stern and the Wharton School, forthcoming *Review of Financial Studies*.

DRIESSEN, J., P. KLAASSEN, AND B. MELENBERG (2003): "The performance of multi-factor term structure models for pricing and hedging caps and swaptions," *Journal of Financial and Quantitative Analysis*, 38:635–672.

DUAN, J.-C. AND J.-G. SIMONATO (1999): "Estimating and testing eksponential-affine term structure models by Kalman filter," *Review of Quantitative Finance and Accounting*, 13:111–135.

DUARTE, J. (2008): "The causal effect of mortgage refinancing on interest rate volatility: Empirical evidence and theoretical implications," *Review of Financial Studies*, 21:1689–1731.

DUARTE, J., F. LONGSTAFF, AND F. YU (2007): "Risk and return in fixed income arbitrage: Nickels in front of a steamroller," *Review of Financial Studies*, 20:769–811.

ERAKER, B. AND I. SHALIASTOVICH (2008): "An equilibrium guide to designing affine pricing models," *Mathematical Finance*, 18:519–43.

Fan, R., A. Gupta, and P. Ritchken (2003): "Hedging in the possible presence of unspanned stochastic volatility: Evidence from swaption markets," *Journal of Finance*, 58:2219–2248.

Filipovic, D. and A. Trolle (2010): "The term structure of basis spreads," Working paper, Ecole Polytechnique Federale de Lausanne.

Fujii, M., Y. Shimada, and A. Takahashi (2009): "A market model of interest rates with dynamic spreads in the presence of collateral and multiple currencies," Working paper, Shinsei Bank and University of Tokyo.

Gallmeyer, M., B. Hollifield, and S. Zin (2005): "Taylor rules, McCallum rules, and the term structure of interest rates," *Journal of Monetary Economics*, 52:921–950.

Gârleanu, N., L. H. Pedersen, and A. M. Poteshman (2009): "Demand-based option pricing," *Review of Financial Studies*, 22:4259–4299.

Ghysels, E., P. Santa-Clara, and R. Valkanov (2005): "There is a risk-return trade-off after all," *Journal of Financial Economics*, 76:509–548.

——— (2006): "Predicting volatility: Getting the most out of return data sampled at different frequencies," *Journal of Econometrics*, 131:59–95.

Giordani, P. and P. Soderlind (2003): "Inflation forecast uncertainty," *European Economic Review*, 47:1037–1059.

Gupta, A. and M. Subrahmanyam (2005): "Pricing and hedging interest rate options: Evidence from cap-floor markets," *Journal of Banking and Finance*, 29:701–733.

Han, B. (2007): "Stochastic volatilities and correlations of bond yields," *Journal of Finance*, 62:1491–1524.

Heidari, M. and L. Wu (2003): "Are interest rate derivatives spanned by the term structure of interest rates?," *Journal of Fixed Income*, 13:75–86.

Heston, S. (1993): "A closed form solution for options with stochastic volatility," *Review of Financial Studies*, 6:327–343.

Jamshidian, F. (1997): "Libor and swap market models and measures," *Finance and Stochastics*, 1:293–330.

Jarrow, R., H. Li, and F. Zhao (2007): "Interest rate caps 'smile' too! But can the LIBOR market models capture the smile?," *Journal of Finance*, 62:345–382.

Joslin, S. (2007): "Pricing and hedging volatility risk in fixed income markets," Working paper, MIT.

Joslin, S., M. Priebsch, and K. Singleton (2010): "Risk premiums in dynamic term structure models with unspanned macro risks," Working paper, MIT and Stanford.

KRISHNAMURTHY, A. (2010): "How debt markets have malfunctioned in the crisis," *Journal of Economic Perspectives*, 24:3–28.

LE, A. AND K. SINGLETON (2010): "An equilibrium term structure model with recursive preferences," *American Economic Review, Papers and Proceedings*.

LEE, R. (2004): "Option pricing by transform methods: Extensions, unification, and error control," *Journal of Computational Finance*, 7:51–86.

LI, H. AND F. ZHAO (2006): "Unspanned stochastic volatlity: Evidence from hedging interest rate drivatives," *Journal of Finance*, 61:341–378.

——— (2009): "Nonparametric estimation of state-price densities implicit in interest rate cap prices," *Review of Financial Studies*, 22:4335–4376.

LONGSTAFF, F., P. SANTA-CLARA, AND E. SCHWARTZ (2001a): "The relative valuation of caps and swaptions: Theory and evidence," *Journal of Finance*, 56:2067–2109.

——— (2001b): "Throwing away a billion dollars: the cost of suboptimal exercise strategies in the swaptions market," *Journal of Financial Economics*, 62:39–66.

LONGSTAFF, F. AND E. SCHWARTZ (2001): "Valuing American options by simulation: A simple least-square approach," *Review of Financial Studies*, 14:113–147.

MERCURIO, F. (2009): "Interest rates and the credit crunch. New formulas and market models," Working paper, Bloomberg.

NELSON, C. AND A. SIEGEL (1987): "Parsimonious modeling of yield curves.," *Journal of Business*, 60:473–489.

PIAZZESI, M. AND M. SCHNEIDER (2007): "Equilibrium yield curves," in, Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, eds.: *NBER Macroeconomics Annual 2006*, MIT press, Cambridge MA.

PITERBARG, V. (2010): "Funding beyond discounting: Collateral agreements and derivatives pricing," *RISK magazine*, pages 97–102.

SHALIASTOVICH, I. (2009): "Learning, confidence and option prices," Working paper, The Wharton School, University of Pennsylvania.

SMITH, J. AND J. TAYLOR (2009): "The term structure of policy rules," *Journal of Monetary Economics*, 56:907–919.

TROLLE, A. B. AND E. S. SCHWARTZ (2009): "A general stochastic volatility model for the pricing of interest rate derivatives," *Review of Financial Studies*, 22:2007–2057.

XIONG, W. AND H. YAN (2010): "Heterogeneous expectations and bond markets," *Review of Financial Studies*, 23:1433–1466.